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**ORBITAL FLIGHT AND COVERAGE SIMULATION**

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR- 64-156

DECEMBER 1964

L. E. Wilkie and J. W. Stevens

Prepared for

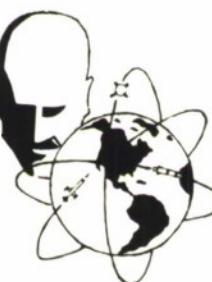
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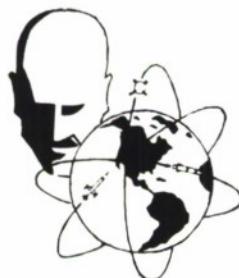
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## FOREWORD

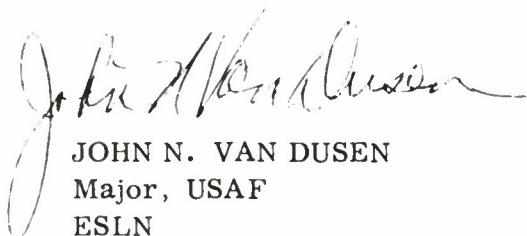
This report is the second in a series; the first report is ESD-TDR-64-112, "Engineering Simulation of Powered Flight," by R. W. Dix, The MITRE Corporation, July 1964.

## ABSTRACT

This report is the second of a series describing the current effort towards establishing a workable engineering simulation of the space-ground environment. This report describes the simulation of ground station coverage of a vehicle from the end of powered flight to an arbitrary time in orbit. Signal strengths of several vehicle-ground station antenna combinations, as well as geometrical coverage, are simulated. A spherical earth model was used in the simulation, and the effects of atmospheric drag and orbital perturbations were neglected.

## REVIEW AND APPROVAL

Publication of this technical documentary report does not constitute Air Force approval of the reports findings or conclusions. It is published only for the exchange and stimulation of ideas.



JOHN N. VAN DUSEN  
Major, USAF  
ESLN

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## SECTION I

### INTRODUCTION

This report is the second in a series describing the current effort toward establishing a workable engineering simulation of the space-ground environment. The activity divides into three phases:

Phase 1: Powered-flight and free-flight trajectory simulation.

Phase 2: Simulation of the interactions between the space vehicle, other vehicles, and the ground environment, such as tracking coverage, telemetry, communications, etc.

Phase 2: Engineering utilization of the above simulations in the solution of a particular problem, such as evaluating or optimizing the communication capacity of a given system, or evaluating the ground environment in relation to a given set or type of missions.

The powered-flight trajectory simulation of Phase 1 has been documented in ESD-TDR-64-112. The free-flight trajectory simulation of Phase 1 and most of Phase 2 is documented in this report. More specifically, the program described in this report calculates the orbital history of a space vehicle from the point of injection to a selected time in the future, the tracking coverage of that vehicle by an arbitrary number of ground stations from launch through orbital history, and the electromagnetic signal strengths of various space-vehicle ground-station antenna configurations.

## PROGRAMMING APPROACH

The engineering simulation of the space-ground environment was developed as a series of programs, each of which performs a specific engineering function. This approach allows considerable flexibility and adaptability to new situations. The over-all simulation is presently separated into five programs which are related as shown in Fig. 1. One program (MAIN) provides executive-type control and acts as the main calling routine. The other four programs (TRAC, RUNG, CALC, DEBE) perform specific engineering simulations and are called subroutines. The programs are described in Section 2 under the heading General Description.

## TYPICAL APPLICATION

This program was used in the study of emergency communication vehicles. Various trajectories were simulated, and the trade-off between payload weight (i.e., transmitted power) and the number and distribution of the ground stations that could communicate with the payload were evaluated. It was possible to quickly arrive at a trajectory and payload weight which satisfied the operational requirements of the systems. The program was also used to evaluate the available tracking coverage for the flight-test trajectory.

## ORGANIZATION OF THE REPORT

The report has been arranged with the interests of the reader in mind. An effort was made to include enough information in Section 2 to permit someone with a general knowledge of digital simulation to use the program without further reading. If a deeper understanding of the simulation is desired, the reader will find the mathematical derivations corresponding to each of the subprograms in the remaining sections. The appendixes contain a complete set of Fortran IV listings.

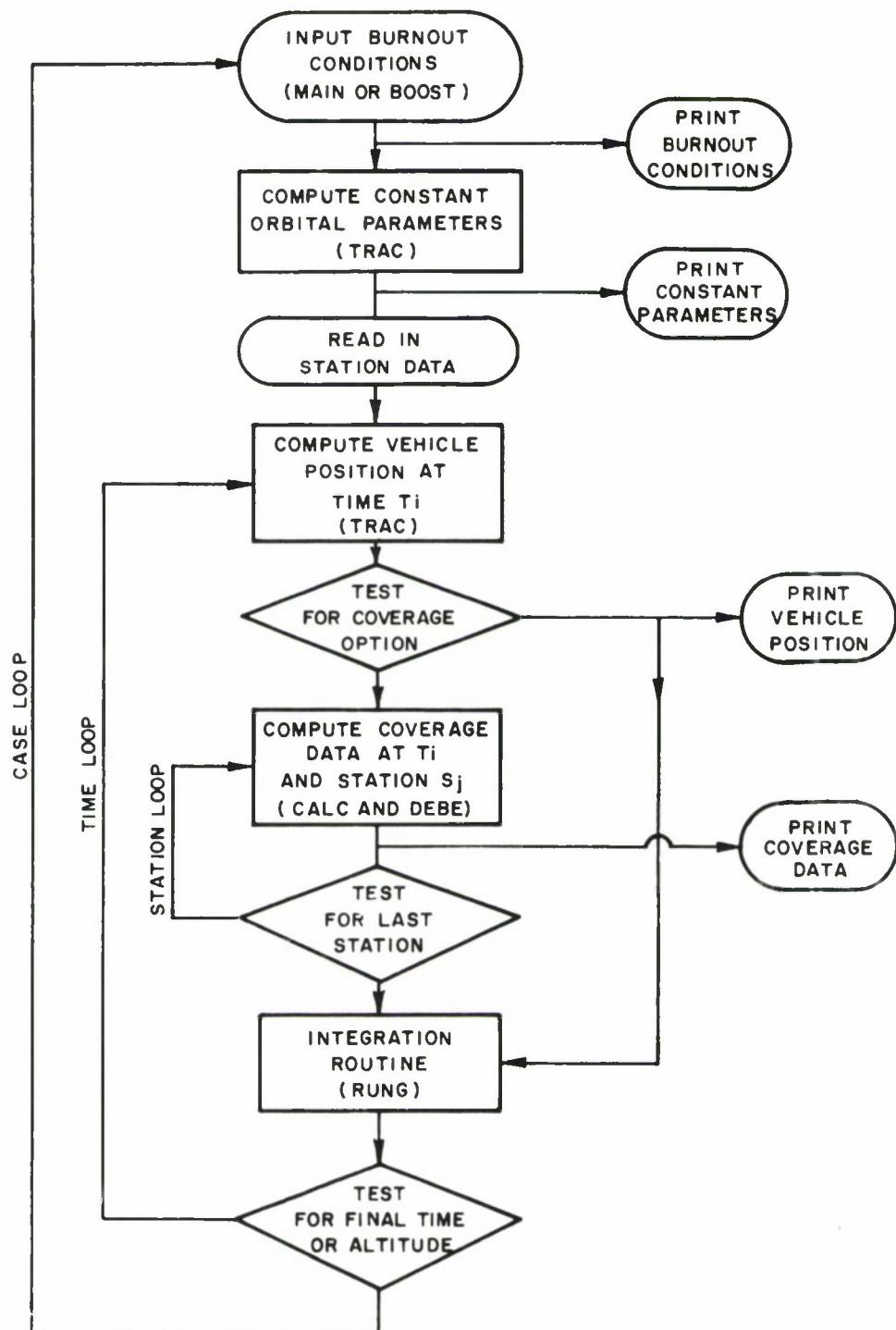


Fig. 1 Functional Flow Diagram

## SECTION II

### USER INFORMATION

The program described in this report assumes a spherical-earth model; the effects of earth perturbations and atmospheric drag on the orbit are neglected. As originally conceived, this program was used in the study of emergency communication rocket systems involving high-loft, suborbital trajectories. Since trajectory times were relatively short, earth perturbation effects could be neglected; considerable programing time was saved by assuming a spherical-earth model. For orbital trajectories involving longer periods of time, earth-perturbation effects cannot, in general, be neglected. The user must decide if these effects are important in his particular application.

#### GENERAL DESCRIPTION

The detailed interrelation of the five programs is shown in the flow chart given in Fig. 2.

Program MAIN acts as the executive or main calling routine. The parameters defining the vehicles position, velocity, and attitude at burnout of the final stage of the booster are read into the program. Subroutine TRAC is called. MAIN also provides a logical Fortran end of program if the burnout velocity is negative. This provision is a programing formality and normally is never used. A normal exit occurs when the input data have all been read in.

Subroutine TRAC calculates the orbital position relative to axes fixed in a spherical, rotating earth. It also controls the calling of the other subroutines. If station coverage is required, CALC is called; if station data is not desired, the

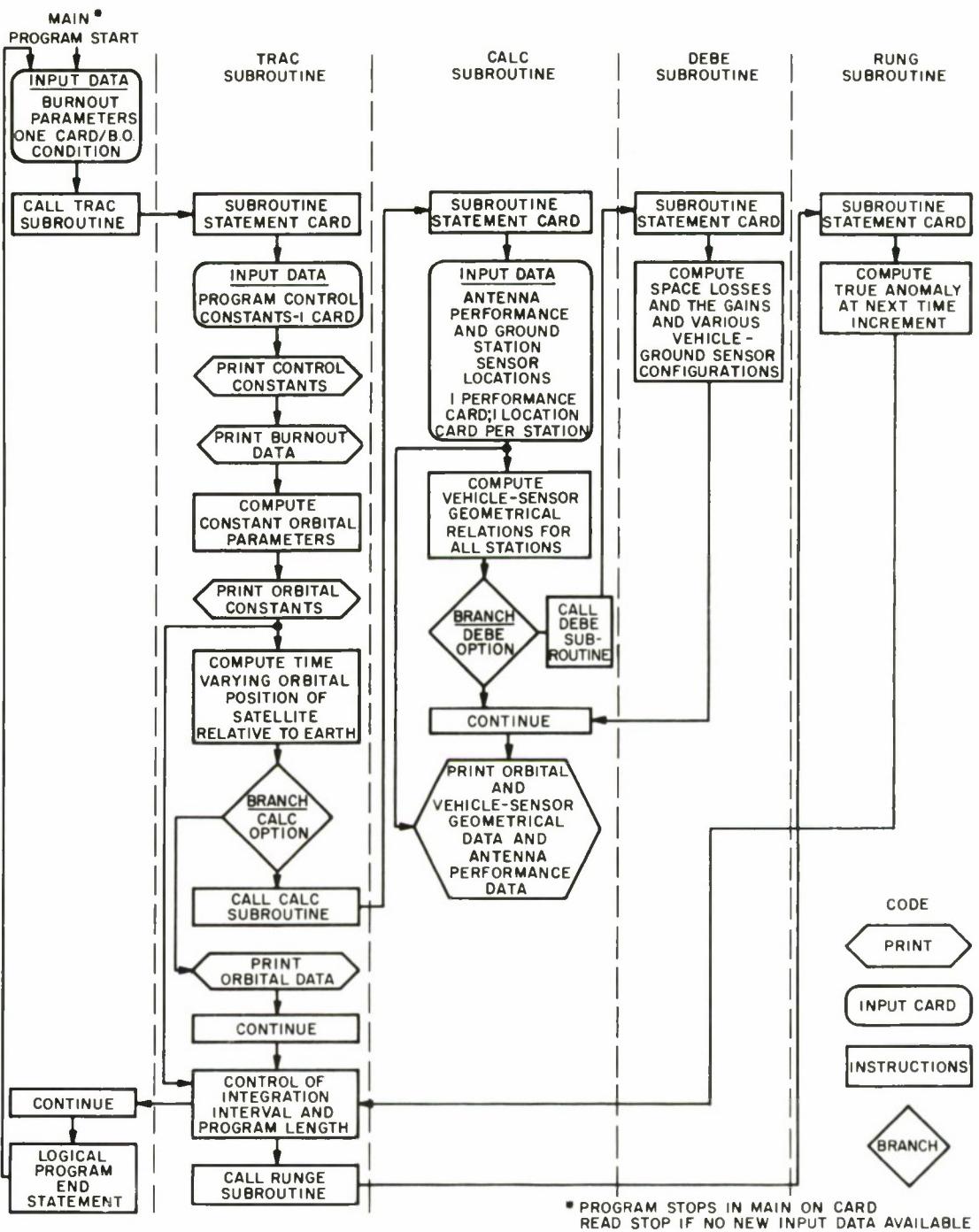


Fig. 2 Functional Flow Diagram--Basic Orbit and Sensor Coverage Program

time-varying orbital data is printed out in TRAC. The basic control data card for this option and other options, time-interval controls, and integration sequencing is read in by TRAC. These control constants are defined below in the description of input data cards.

Subroutine CALC performs, in general terms, the following functions:

- (1) Reads tracking station geographic and antenna data. Up to 150 stations are permissible.
- (2) Reads vehicle and sensor antenna performance data.
- (3) Calculates geometrical relations between instantaneous vehicle position and stations.
- (4) Calculates rates of change of the above relations by time averages.
- (5) Calculates angles defining several vehicle-sensor antenna axis orientations.
- (6) Calls DEBE option.
- (7) Prints out orbital and station coverage data.

Subroutine DEBE calculates antenna performance and space losses for eight combinations of vehicle and ground antenna configurations. The vehicle antenna may be either inertially fixed (spin stabilized), or constantly pointing towards the center of the earth (earth centered), or it may be tracking the ground stations (vehicle tracking). The ground antenna may be fixed with a specified beamwidth (ground fixed), or continually tracking the vehicle with a specified beamwidth (ground tracking), or it may be isotropic. There are nine possible combinations of these antennas, but the vehicle-tracking isotropic-ground case is not computed here. However, the performance of this combination can be

obtained as a special case of the vehicle-tracking fixed-ground case by choosing the proper beamwidth for the ground antenna. Space losses are also printed out separately.

In subroutine RUNG a numerical integration of the true anomaly equation is performed by a fourth-order Runge-Kutta method.

There are no READ or PRINT statements in either DEBE or RUNG.

Subroutines TRAC and RUNG together solve the equation of motion of the vehicle and generate the ephemeris data for the vehicle. The use of CALC and DEBE is optional depending on the application under investigation. Thus, the over-all program is made up of engineering building blocks which can be put together to solve a particular problem with economy of computer time and a minimum of extraneous information in the output. A further advantage of this approach is that refinements and additions can be incorporated into the individual subroutines without the necessity of recompiling the whole program.

#### INPUT DATA CARDS

The proper sequencing of cards for the running of multiple cases is described under the heading Stacking Cases. The program is set up to determine the orbital coverage capabilities of a given arbitrary set of global tracking stations for n-different orbit configurations. The number of input data cards in any given run will depend on both the number of stations and the number of orbits investigated. However, there are only four basic types of input data cards.

Card #1 contains the burnout data which establishes the orbit. Read-in occurs in program MAIN. There will be one such card for each orbit configuration investigated. The second to n-sets of burnout data cards are arranged sequentially at the end of data input deck.

Card #2 contains the control constants which are used by the program. These constants control the integration interval, printout intervals and increments, and the subroutine and the time-stop options. The printout interval is the subdivision of time in which the data is printed out within the larger printout increment. The purpose of the printout increment option is to allow for a "fine-grained" printout during portions of the trajectory that are changing rapidly (or are of more interest than some other part of the trajectory), and conversely, to allow a "coarse-grained" printout during slowly varying parts of the trajectory. The unused control constants were inserted for use in future program modifications. Only one such card will appear in any given machine run. Read-in occurs in subroutine TRAC.

Card #3 contains ground and vehicle sensor data. Only one such card will appear in any given machine run. Read-in occurs in subroutine CALC.

Card #4 contains the geographic location, minimum antenna elevation angle, and beamwidth data of the tracking stations. There will be one card for each station, and these cards will be arranged sequentially when more than one station is being investigated. Read-in occurs in subroutine CALC.

The card formats and Fortran nomenclature are summarized in Tables 1, 2, 3, and 4.

#### STACKING CASES

For runs with one orbit configuration and multiple ground tracking stations, the proper data card stacking is:

Card #1, burnout conditions

Card #2, control constants

Card #3, vehicle and ground sensor data

Table 1

Burnout Data

CARD #1 (One Card)

Fortran Variable	Card Column Location	Fortran Numeric Format	Definition
HBO	1-10	F10.2	Burnout altitude - measured positive UP from surface of earth (NM)
VBO	11-20	F10.2	Burnout velocity (FT/SEC)
GBO	21-30	F10.2	Burnout flight path angle - measured positive up from local horizontal plane (DEG)
BOHEAD	31-40	F10.2	Burnout heading - measured positive clockwise from north (DEG)
BOLAT	41-50	F10.2	Burnout latitude - measured positive North from the Equator (DEG)
BOLONI	51-60	F10.2	Burnout longitude - measured positive East from Greenwich (DEG)
BIG	61-70	F10.2	Burnout spin axis attitude - measured positive up from local horizontal plane (DEG)
BIH	71-80	F10.2	Burnout spin axis heading - measured positive clockwise from North (DEG)

Table 2  
Program Control Constants

Card #2 (One Card)

Fortran Variable	Card Column Location	Fortran Numeric Format	Definition
NUM (1)	1-5	I5	Number of ground sensors (maximum of 150 sensors)
NUM (2)	6-10	I5	Number of integration intervals in the initial and final printout increment
NUM (3)	11-15	I5	End of first printout increment in seconds
NUM (4)	16-20	I5	Number of integration interval in intermediate printout increment
NUM (5)**	21-25	I5	Final time in seconds or as modified by NUM (16)
NUM (6)	26-30	I5	Control constant to select time stop option ≤ 0 bypass option - use increments > 0 select option
NUM (7)	31-35	I5	Control constant to select CALC- subroutine option ≤ 0 bypass option - use > 0 select option
NUM (8)	36-40	I5	Integration interval in seconds
NUM (9)	41-45	I5	Control constant to select DEBE - subroutine option > 0 bypass option ≤ 0 select option
NUM (16)*	76-80	I5	Constant which multiplies NUM (5) to extent final time; i. e., when NUM (16) = 1 , NUM (5) is in sec. NUM (16) = 60 , NUM (5) is in min. NUM (16) = 3600, NUM (5) is in hrs, etc.

\*NOTE: NUM (10) through NUM (15) are unassigned.

\*\*NOTE: In the program printout of the program control constants NUM (5) reverts to the total program time in seconds.

Table 3

Sensor Data - Ground and Vehicle

Card #3 (One Card)

Fortran Variable	Card Column Location	Fortran Numeric Format	Definition
FREQ	1-10	F10.2	Antenna operating frequency in megacycles
BW1	11-20	F10.2	Beamwidth of spaceborne antenna measured between half-power points in degrees

Table 4  
Station Data

Card #4 (NUM (1) Cards\*)

Fortran Variable	Card Column Location	Fortran Format	Definition
	9-24	2A8	These 16 characters reserved for the station name
ALAT (I)	25-32	F8.2	Latitude of the Ith ground sensor measured positive north of equator (Deg.)
ALONG (I)	33-40	F8.2	Longitude of the Ith ground sensor - measured positive east of Greenwich meridian (Deg.)
ANEL (I)	41-47	F8.2	Elevation of the Ith ground sensor, if fixed - measured positive above tangent plane (Deg.)
ANAZ (I)	48-56	F8.2	Azimuth of the Ith ground sensor, if fixed - measured positive clockwise from north (Deg.)
ALTS (I)	57-64	F8.2	Altitude of station above seal level (feet)
BW (I)	65-72	F8.2	Beamwidth of the Ith ground sensor - measured between half-power points (Deg.)
ELEM (I)	73-80	F8.2	Minimum elevation angle of tracking antenna (Deg.)

---

\* One of these cards is required for each station. The Fortran variable NUM (1) must coincide with the number of station cards. The number of stations can vary from 1 to 150.

Card #4, station 1

Card #4, station n

For runs with more than one orbit configuration and ground tracking station, the proper data card stacking is

Card #1, first burnout conditions

Card #2, control constants

Card #3, vehicle and antenna sensor data

Card #4, station 1

Card #4, station 2

Card #4, station n

Card #1, second burnout condition

Card #1, mth burnout conditions

## OUTPUTS

In the most general case, when all functional options are selected, the program delivers the following five types of output data: burnout conditions, orbital parameters at burnout, station data, vehicle and sensor parameters, and coverage data.\* The coverage data includes the vehicle ephemeris data and the performance data for each station that is in the geometrical line of sight of the

---

\* For convenience in checking the options that have been selected, the program control constants as defined in Table 2 appear at the beginning of the printout.

vehicle as a function of time. If the CALC option is not selected, only the vehicle burnout and ephemeris data will be printed. The definition of each variable in the five groups as it appears on the printed output is described in Tables 5 through 9. A typical printout is shown in Table 10.

Table 5

Burnout Conditions

Output #1\*

Variable	Units	Definition
ALTITUDE	n. m.	Burnout altitude from the surface of the earth
VELOCITY	ft/sec	Inertial burnout velocity
GAMMA	deg.	Inertial burnout flight path angle
HEADING	deg.	Inertial heading at burnout
LATITUDE	deg.	Vehicle latitude at burnout
LONGITUDE	deg.	Vehicle longitude at burnout
SPIN AXIS ATTITUDE	deg.	Antenna axis attitude at burnout
SPIN AXIS HEADING	deg.	Inertial heading of the antenna axis at burnout

---

\* These outputs correspond exactly to the burnout data read in on card #1 (see Table II).

Table 6

Orbital Parameters At Burnout

Output #2

Variable	Units	Definition
ECCENTRICITY	none	Eccentricity of the orbit
INCLINATION	deg.	Inclination of the orbital plane $< 90^\circ$ prograde, $> 90^\circ$ retrograde
SEMI LAT REC	n. m.	Semi-latus rectum
TRUE ANOMALY	deg.	Angle between perigee and the vehicle in the orbital plane at burnout
OMEGA	deg.	Angle between ascending node and perigee in the orbital plane at burnout
ASNODE	deg.	Angle between the ascending node and Greenwich (+ East)

Table 7  
Station Data

Output #3\*

Variable	Units	Definition
NUMBER	none	Station number
NAME	none	Station name limited to 16 characters
LAT	deg.	Station latitude (+ north of the Equator)
LONG	deg.	Station longitude (+ east of the Greenwich meridian)
ELEV	deg.	Elevation angle of the fixed ground sensor axis above the tangent plane
AZIMUTH	deg.	Azimuth of the fixed ground sensor axis (+ clockwise from north)
ALTITUDE	feet	Altitude of the station above sea level
BEAMWIDTH	deg.	Beamwidth of the ground sensor measured between half power points
MIN ELEV ANGLE	deg.	Minimum elevation angle of tracking antenna

---

\* These outputs correspond to the station data read in on card #4 (one for each station) (see Table 4).

Table 8  
Vehicle And Sensor Parameters

Output #4

Variable	Units	Definition
BEAMWIDTH	deg.	Vehicle antenna beamwidth measured between the half-power points
TRACKING GAIN	db	Vehicle on axis antenna gain
OPERATING FREQUENCY	megacycles	Operating frequency of the sensors

NOTE: The definition of the losses for the various vehicle-ground antenna configurations appears following the above output for easy reference

Table 9  
Coverage Data

Output #5 Part I - Vehicle Position

Variable	Units	Definition
TIME	min.	Instantaneous time in orbit
TIME	hrs.	Instantaneous time in orbit
LATITUDE	deg.	Vehicle latitude in orbit
ALTITUDE	min.	Vehicle altitude above the surface of the earth
TRUE ANOMALY	deg.	True anomaly
OMEGA	deg.	Angle between perigee and ascending node in the orbital plane
ASNODE	deg.	Angle between the ascending node and the Greenwich meridian

Table 9 (Cont'd)

Part II - Stations in Contact

Variable	Units	Definition
ELEVATION*	deg.	Elevation angle of the line of sight from the station to the vehicle (+ up from the tangent plane)
AZIMUTH*	deg.	Azimuth angle of the line of sight between the station and the vehicle
RANGE*	n. m.	Line of sight distance between the station and the vehicle
ANT ANGLE* SPIN STAB	deg.	Angle between spin-stabilized vehicle antenna and the line of sight between the station and the vehicle
ANT ANGLE* SPIN STAB	deg.	Angle between the earth-centered vehicle antenna and the line of sight between the station and the vehicle
GRD ANT* ANGLE	deg.	Angle between the fixed ground antenna axis and the line of sight between the station and the vehicle
VEHICLE ANT GAINS	db	Gains of the space-borne antennas
SPIN STAB	db	Spin-stabilized antenna
VERT-STAB	db	Earth-centered antenna gain
GRD ANT GAIN	db	Gain of the fixed ground antenna
SPACE LOSS	db	Total space loss
DB1 through DB8	db	Total losses for the various vehicle ground antenna configurations. Definitions appear on each printout (see DEBE listing in Appendix E)

NOTE: The rate of change of each of the parameters marked with an asterisk is printed out directly below the value of each of the parameters.

Table 10

Typical Output FormatSpace Vehicle Tracking and Sensor Coverage Program

PROGRAM CONTROL CONSTANTS									
NUM(1)	NUM(2)	NUM(3)	NUM(4)	NUM(5)	NUM(6)	NUM(7)	NUM(8)	NUM(9)	NUM(10)
6	1	1	1	1	1	1	1	1	1
600	600	600	600	600	600	600	600	600	600

ORBITAL CONDITIONS									
ALTITUDE	VELOCITY	CAPPA	HEADING	LATITUDE	LONGITUDE	SPIN AXIS ATTITUDE	SPIN AXIS HEADING	AS NODE	DEG
N MI	FT/SEC	DEG	DEG	DEG	DEG	DEG	DEG	DEG	0.00
200.0	25234.42	0.73	106.36	28.48	-80.61	0.00	0.00	0.00	0.00

ORBITAL PARAMETERS AT BURNOUT									
ECCENTRICITY	INCLINATION	SMA	LAT REC	TRUE ANOMALY	DMEGA	AS NODE	DEG	DEG	DEG
0.00001	32.50	3638.15	N MI	180.00	-62.56	157.783	0.00	0.00	0.00

STATISTICS									
NUMBER	NAME	LAT	LONG	ELEV	AZIMUTH	ALTITUDE	FLAT WIDTH	MIN ELEV ANGLE	DEG
1	CAPE KENNEDY	28.45	-80.61	0.00	0.00	0.00	16.30	0.00	0.00
2	VALKARIA	27.95	-76.52	0.00	0.00	0.00	16.30	0.00	0.00
3	ELEUTHERA	25.25	-76.52	0.00	0.00	0.00	16.30	0.00	0.00
4	SAN SALVADOR	24.12	-74.51	0.00	0.00	0.00	16.30	0.00	0.00
5	ANTIGUA	17.14	-61.19	0.00	0.00	0.00	16.30	0.00	0.00
6	GR CANARY IS	27.74	-15.00	0.00	0.00	0.00	16.30	0.00	0.00

VEHICLE AND SENSOR PARAMETERS									
BEAM WIDTH	TRACKING GAIN	OPERATING FREQUENCY	DEG	DEC	MC	MC	MC	MC	MC
50.0	11.21	243.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

**TOTAL LOSSES FOR VARIOUS VEHICLE-GROUND ANTENNA CONFIGURATIONS**

DB1=VEHICLE SPIN STABILIZED-GROUND FIXED  
 DB2=VEHICLE SPIN STABILIZED-GROUND TRACKING  
 DB3=VEHICLE EARTH CENTERED-GROUND TRACKING  
 DB4=VEHICLE EARTH CENTERED-GROUND FIXED  
 DB5=VEHICLE TRACKING-GROUND FIXED  
 DB6=VEHICLE TRACKING-GROUND TRACKING  
 DB7=VEHICLE SPIN STABILIZED-ISOTROPIC GROUND  
 DB8=VEHICLE EARTH CENTERED-ISOTROPIC GROUND

Table 10 (Continued)

VEHICLE POSITION	TIME	LATITUDE	LONGITUDE	ALTITUDE	TRUE ANGULAR	UMEGA	ASNDDE
MINUTES	HOURS	DEG	DEG	DEG	DEG	DEG	DEG
<b>1</b>							
STATIONS IN CONTACT	ELEVATION	AZIMUTH	RANGE	ANT ANGLE	GRD ANI	GRD ANI	VEHICLE-GROUND ANTENNA LOSSES
	DEG	DEG	FEET	SPIN-STAB	ANGLE	GAINS DE	DB
	DEG/SEC	DEG/SEC	FEET/SEC	DEC	DEC	SPIN-STAB	DB3
CAPE KENNEDY	39.22	107.44	3421	103.332	47.6	103.46	-9.90
	.8463	.2057	10552.7	.2222	.7844	.2237	-3.21
VALKARIA	40.84	99.58	29535	97.461	45.64	97.23	-5.91
	.6567	-4.2144	5381.7	.2748	.689	.2798	-1.85
ELEUTHRA	56.50	357.73	237.82	59.764	31.44	56.53	-13.33
	.4216	.7546	-12531.7	.1256	.3749	.254	6.20
SAN SALVADOR	39.27	329.75	3.95	53.196	47.82	48.03	-13.15
	.2985	.3353	-17325.3	.772	.2439	.6877	-3.17
ANTIGUA	2.15	309.18	106.60	60.850	70.80	51.05	-11.67
	.5613	.0431	-21962.2	.282	.610	.428	-3.60
<b>2</b>							
STATIONS IN CONTACT	ELEVATION	AZIMUTH	RANGE	ANT ANGLE	GRD ANI	GRD ANI	VEHICLE-GROUND ANTENNA LOSSES
	DEG	DEG	FEET	SPIN-STAB	ANGLE	GAINS DE	DB
	DEG/SEC	DEG/SEC	FEET/SEC	DEC	DEC	SPIN-STAB	DB3
CAPE KENNEDY	19.81	107.55	499.55	106.261	62.76	106.48	-13.11
	.3225	.0020	19822.8	.488	.2616	.051.	-8.91
VALKARIA	20.53	103.77	468.00	102.809	62.25	102.88	-9.44
	.3304	.0699	15516.0	.6891	.2768	.0941	-9.54
ELEUTHRA	40.91	76.64	254.57	80.426	45.28	75.54	-2.06
	.2598	-4.6849	5868.1	.3444	.256	.3562	-1.79
SAN SALVADOR	51.15	40.26	252.40	63.770	36.36	61.40	-7.82
	.1960	-4.8249	-5221.1	.1762	.1777	.2228	4.16
ANTIGUA	6.71	313.64	83.74	58.351	69.81	47.33	-18.49
	.0761	.0642	-21656.8	.417	.165	.507	-3.33

Table 10 (Continued)

VEHICLE POSITION	TIME MINUTES	TIME HOURS	LATITUDE DEG	LONGITUDE DEG	ALTITUDE NM	TRUE ANGULARITY DEG	OMEGA DEG	ASCEND DEG	
CAPE KENNEDY		24.612 .05	24.612 .05	-68.516	200.20	191.73	-62.561	157.783	
STATIONS IN CONTACT	ELEVATION DEG/SEC	ELEV-RATE DEG/SEC	AZIMUTH DEG	RANGE DEG NAUT MI FEET/SEC	ANT ANGLE VERT-STAB DEG SEC	GR C ANT ANGLE VERT-STAB DEG SEC	GAINS DB SPIN-STAT VERT-STAB	GRD ANT GAIN DB SPACE	VEHICLE-GROUND ANTENNA LOSSES
VALKARIA	105.20 .0239	105.20 .0239	93.76	7.242	104.632 .0304	68.07 .0970	104.52 .0341	-11.17 .475	-160.53 .142.45
ELEUTHRA	93.76 .2854	93.76 .2854	83.45	17899.5	92.507 .2013	61.50 .2654	93.50 .2255	-3.89 .16.60	-129.68 .129.68
SAN SALVADOR	83.45 .7199	83.45 .7199	83.604	12248.1	83.604 .3125	55.13 .3125	84.32 .2821	-2.17 .21.34	-142.64 .142.64
ANTIGUA	83.45 .1033	83.45 .1033	847.86	-2C849.3	847.86 .0648	67.05 .0453	42.19 .0856	-17.63 .529	-159.54 .141.74
VEHICLE POSITION	TIME MINUTES	TIME HOURS	LATITUDE DEG	LONGITUDE DEG	ALTITUDE NM	TRUE ANGULARITY DEG	OMEGA DEG	ASCEND DEG	
CAPE KENNEDY		107.13 .0119	107.13 .0119	22,668.3	200.20	195.657	-62.561	157.783	
STATIONS IN CONTACT	ELEVATION DEG/SEC	ELEV-RATE DEG/SEC	AZIMUTH DEG	RANGE DEG NAUT MI FEET/SEC	ANT ANGLE VERT-STAB DEG SEC	GR C ANT ANGLE VERT-STAB DEG SEC	GAINS DB SPIN-STAT VERT-STAB	GRD ANT GAIN DB SPACE	VEHICLE-GROUND ANTENNA LOSSES
VALKARIA	105.9 .0128	105.9 .0128	95.05	522.89	105.44 .0135	70.27 .0366	105.51 .0165	-12.08 .375	-154.38 .144.94
ELEUTHRA	95.05 .1933	95.05 .1933	945.01	22327.3	945.01 .1656	67.70 .1754	95.29 .1195	-5.68 .495	-154.56 .142.16
SAN SALVADOR	95.05 .2121	95.05 .2121	462.22	-18755.6	462.22 -.0985	61.03 -.11010	35.02 -.11195	-4.84 .1135	-149.45 .138.81
ANTIGUA	95.05 .2121	95.05 .2121	48.533	-18755.6	48.533 -.0985	61.03 -.11010	35.02 -.11195	-4.84 .1135	-149.45 .138.81

Table 10 (Continued)

VEHICLE POSITION	TIME MINUTES	TIME HOURS	LATITUDE DEG	LONGITUDE DEG	ATTITUDE ANG. N°	TRUE ANOMALY DEG	UMEGA DEG	ASNUDE DEG
STATIONS IN CONTACT	ELEVATION DEG	AZIMUTH DEG	RANGE DEG	ANT ANGLE DEG	GRC ANT GAINS DB	GRD ANT GAINS DB	VEHICLE-GROUND ANTENNA LOSSES	
	DEG/SEC	DEG/SEC	FEET/SEC	SPIN-STAB DEG/SEC	VERT-STAB DEG/SEC	SPIN-STAB DEG/SEC	VERT-STAB DEG/SEC	
CAPE KENNEDY	•0752	107.89	1158.75	107.294	76.91	107.89	-14.62	-5.76 -167.17 -156.11 -141.35 -161.41
	-0.0713	•0.0718	22544.5	•0.0704	•0.0704	•0.0704	-3.56	-146.79 -137.47 -126.42 -111.65 -150.45
VALKARIA	•075	106.47	1145.21	105.845	70.90	106.47	-12.57	-5.69 -164.96 -155.95 -141.18 -159.27
	-0.0724	•0.0784	22523.8	•0.0767	•0.0765	•0.0765	-3.56	-146.69 -135.33 -126.32 -111.55 -150.25
ELEUTHRA	5.6t	102.36	897.21	100.066	76.12	102.36	-7.40	-19.27 -171.24 -167.64 -152.64 -151.97
	-0.1015	•0.0478	22.13.3	•0.0415	•0.0404	•0.0404	-2.84	-144.57 -128.03 -124.48 -109.43 -148.42
SAN SALVADOR	8.72	59.98	777.46	97.016	69.08	59.86	-5.69	-11.66 -160.68 -159.26 -143.78 -149.01
	-0.1270	•0.0822	21424.2	•0.0705	•0.0668	•0.0636	-4.27	-143.33 -125.08 -123.66 -108.18 -147.60
ANTIGUA	34.59	2.43	334.0	44.827	51.08	34.66	-1.14	-8.87 -146.00 -153.37 -133.66 -137.13
	-0.2062	-5.4962	-12985.3	-0.2618	-0.1659	-0.062	-8.51	-135.99 -113.19 -120.56 -100.85 -144.50
VEHICLE POSITION	TIME MINUTES	TIME HOURS	LATITUDE DEG	LONGITUDE DEG	ATTITUDE ANG. N°	TRUE ANOMALY DEG	UMEGA DEG	ASNUDE DEG
STATIONS IN CONTACT	ELEVATION DEG	AZIMUTH DEG	RANGE DEG	ANT ANGLE DEG	GRC ANT GAINS DB	GRD ANT GAINS DB	VEHICLE-GROUND ANTENNA LOSSES	
	DEG/SEC	DEG/SEC	FEET/SEC	SPIN-STAB DEG/SEC	VERT-STAB DEG/SEC	SPIN-STAB DEG/SEC	VERT-STAB DEG/SEC	
ELEUTHRA	1.24	104.10	1117.02	101.491	70.80	104.10	-8.39	-9.12 -163.99 -159.17 -144.40 -154.87
	-0.0740	•0.0291	22341.5	•0.0237	•0.0216	•0.0201	-3.57	-146.48 -130.93 -126.11 -111.34 -150.05
SAN SALVADOR	3.55	102.71	995.50	99.313	70.60	102.69	-6.93	-15.56 -167.97 -164.70 -149.83 -152.40
	-0.0862	•0.0456	22288.9	•0.0382	•0.0253	•0.0471	-3.67	-145.47 -128.47 -125.21 -110.33 -149.14
ANTIGUA	34.11	52.77	337.50	59.896	51.49	55.54	-13.61	-2.15 -151.84 -147.48 -127.03 -149.69
	-0.0080	•0.0395	355.2	•0.2512	•0.068	•0.4213	-9.25	-136.08 -125.75 -121.39 -100.94 -145.33

Table 10 (Continued)

VEHICLE POSITION	TIME MINUTES	TIME HOURS	LATITUDE DEG	LONGITUDE DEG	ALTITUDE NM	TRUE ANOMALY DEG	OMEGA DEG	ASANODE CEG
STATIONS IN CONTACT	7	.12	17.992	-56.616	260.0C	2E7.47C	-62.561	157.783
ANTIGUA								
VEHICLE POSITION	TIME MINUTES	TIME HOURS	LATITUDE DEG	LONGITUDE DEG	ALTITUDE NM	TRUE ANOMALY DEG	OMEGA DEG	ASANODE CEG
STATIONS IN CONTACT	8	.13	16.145	-51.244	2CC.0C	211.354	-62.561	157.783
ANTIGUA								
VEHICLE POSITION	TIME MINUTES	TIME HOURS	LATITUDE DEG	LONGITUDE DEG	ALTITUDE NM	TRUE ANOMALY DEG	OMEGA DEG	ASANODE CEG
STATIONS IN CONTACT	9	.15	14.238	-47.94C	2CC.0C	215.319	-62.561	157.783
ANTIGUA								

## POWERED FLIGHT OPTION

The complete engineering simulation of the space-ground environment covers the entire mission of the space vehicle from launch through the orbital motion. In many situations, however, the detailed solution from launch is not required. Program MAIN permits the convenient division of the simulation at the point of injection.

The over-all program can be operated in two basic modes:

In the first, or normal mode, the complete solution from launch through orbital motion is obtained, and program MAIN is not used. Burnout conditions are calculated directly in the powered-flight-trajectory program described in Reference [1] and passed on to the proper subroutines which compute orbital position and station coverage.\*

In the second mode, the burnout conditions are not calculated but inserted into the program on input data card #1. Thus, the launch-trajectory portion of the simulation is not used. Program MAIN then functions as the executive program for the reading and routing of these parameters into the subroutines computing orbital position and station coverage. The program also provides for the proper end-of-program statements for the logical Fortran IV termination of the program.

## SPECIAL ROUTINES

There are only two special subroutines required to run the program on the IBM 7030 computer. These are the inverse sine and cosine functions of an angle. These routines were prepared for use with this simulation, and their Fortran IV lists are included in Appendix VI.

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\* The only modification required in the Powered Flight Trajectory Program as described in Reference [1] is the addition of a CALL TRAC (...) statement following the test for last-stage burnout.



### SECTION III

#### ORBITAL SIMULATION - SUBROUTINE TRAC

Subroutine TRAC, called by program MAIN or the powered-flight-trajectory program, has two primary functions. First, it calculates the time-varying orbital position of the vehicle relative to geocentric axes fixed in a spherical rotating earth. Second, it performs the basic control functions of CALC subroutine selection and of integration interval and printout interval management. A functional flow diagram of TRAC is given in Fig. 3 and its Fortran IV listing is given in Appendix II.

#### CALCULATION OF FIXED ORBITAL PARAMETERS

Orbital position determination as a function of time starts with the calculation of the fixed orbit geometry and orbit position relative to the earth from the burnout conditions. The engineering simulation described in this report assumes a spherical-earth model and neglects perturbation effects, such as nodal precession and rotation of the line of perigee in the orbital plane. Consequently, once the burnout conditions are specified, namely, altitude (HBO), velocity (VBO), flight-path angle (BIG), heading (BOHEAD), latitude (BOLAT), and longitude (BOLONG), the position of the orbital plane and the orbit of the vehicle in that plane are completely established and fixed in time. This geometry is completely described by the following fixed orbital parameters which are computed in subroutine TRAC.

- (1) PLAIN - the orbital plane inclination relative to earth.
- (2) VEE - initial true anomaly or the angle between the line of perigee and the position of the vehicle in orbit in the orbital plane at injection.

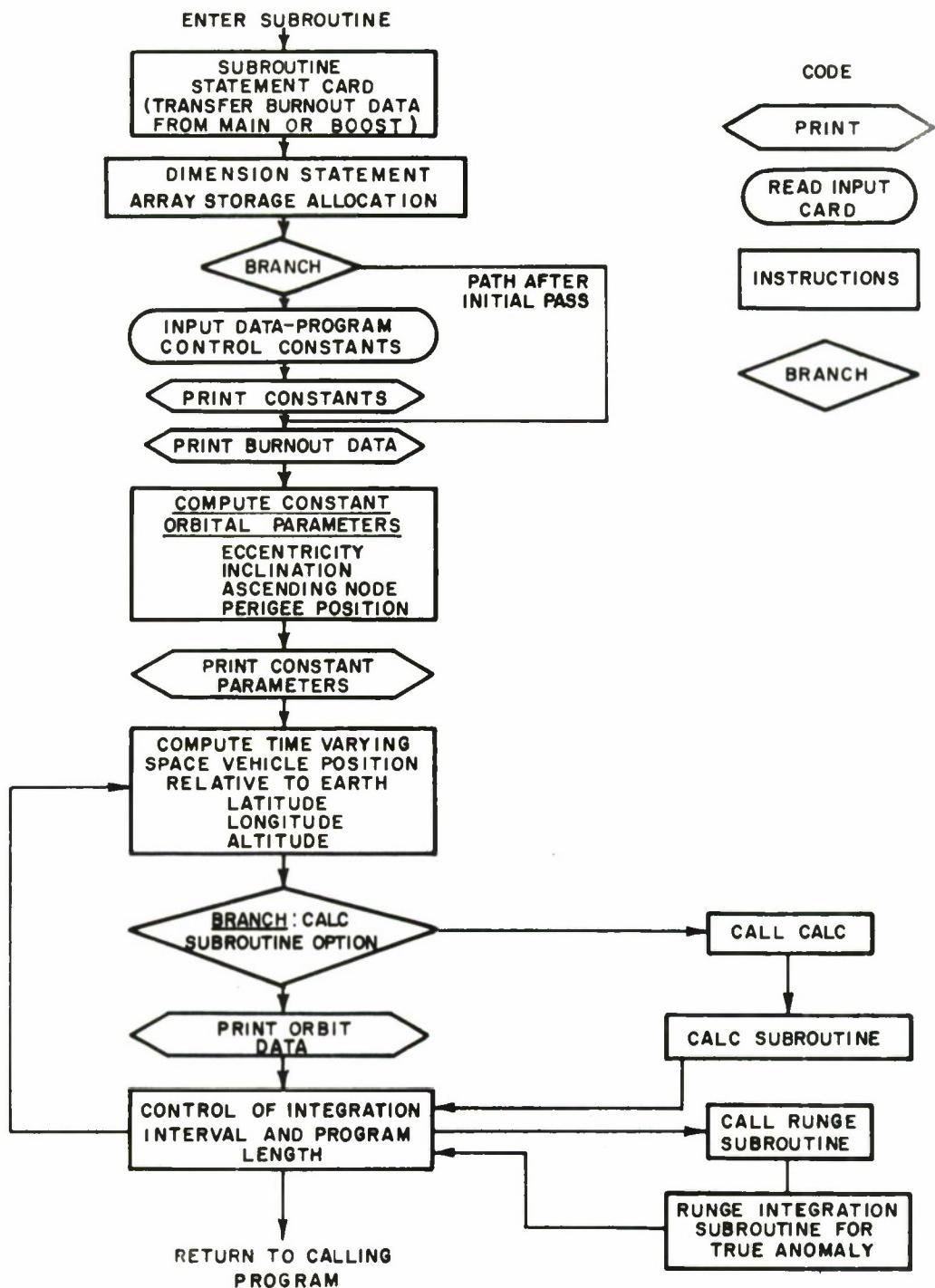


Fig. 3 Functional Flow Diagram--Subroutine TRAC

- (3) ECC - eccentricity of the orbit.
- (4) P - semi-latus rectum.
- (5) OMEGA - perigee position relative to the ascending node in the plane of the orbit.
- (6) ASNODE - longitude of ascending node relative to an earth reference meridian.

These quantities are shown in Fig. 4. The mathematical expressions for each of these quantities are derived below.

#### Inclination Angle (PLAIN)

The inclination angle can be readily determined from the spherical triangle shown in Fig. 4.

From the cosine law

$$\cos \text{PLAIN} = \frac{\cos BC - \cos AC \cos AB}{\sin AC \sin AB} \quad (1)$$

$$\begin{aligned} \cos AB &= \cos BC \cos AC - \sin BC \sin AC \cos C \\ &= \cos DC \cos AC \quad (\text{since } C = 90^\circ) \end{aligned} \quad (2)$$

From the sine law

$$\begin{aligned} \frac{\sin AB}{\sin C} &= \frac{\sin AC}{\sin B} \\ \text{or } \sin AB &= \sin AC / \sin B \quad (\text{since } C = 90^\circ) \end{aligned} \quad (3)$$

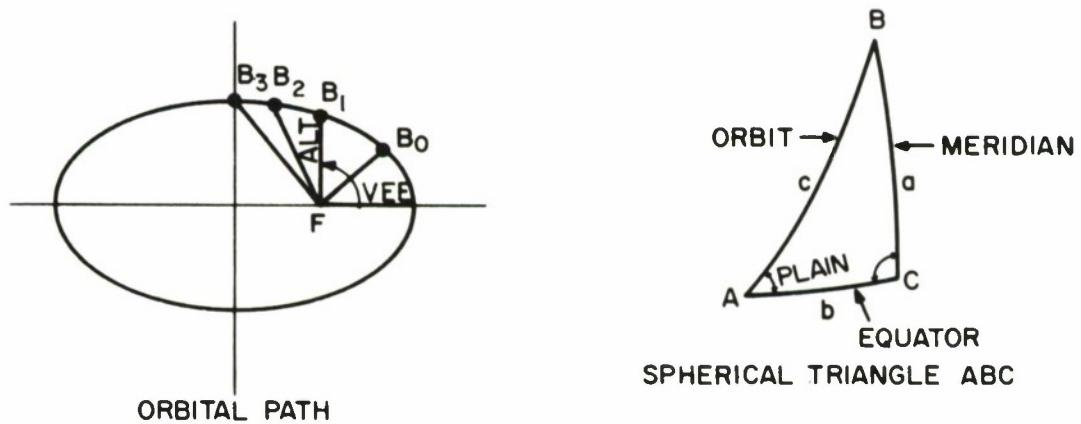
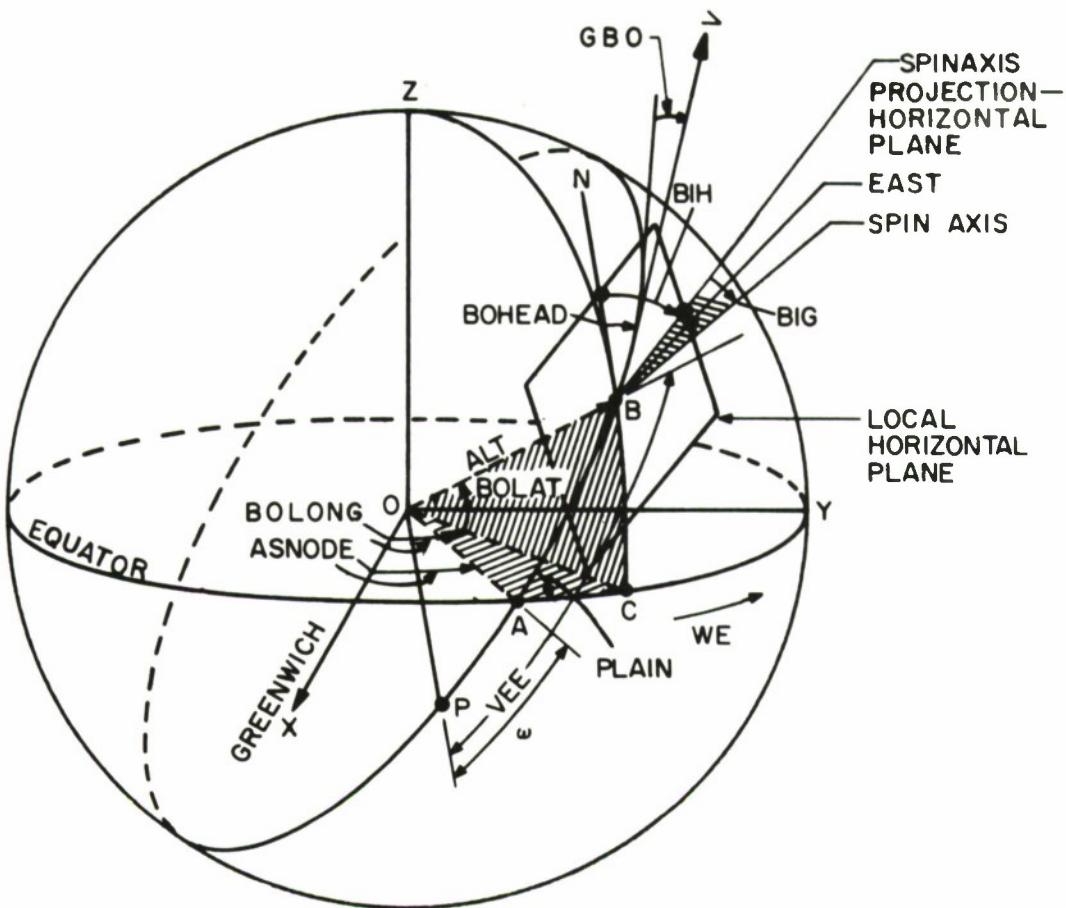


Fig. 4 Orbital Geometry

Substituting Eqs. (2) and (3) in Eq. (1) gives

$$\begin{aligned}\cos(\text{PLAIN}) &= \frac{\cos BC - \cos BC \cos AC \cos AC}{\sin^2 AC / \sin B} \\ &= \frac{\cos BC (1 - \cos^2 AC) \sin B}{\sin^2 AC} \\ &= \cos BC \sin B\end{aligned}\tag{4}$$

where BC is the burnout latitude

B is the burnout heading angle

In Fortran notation

$$\text{PLAIN} = \text{ACOS}(\text{CLABO} * \text{SHEBO})\tag{5}$$

where

\* = multiplication

CLABO = cosine of burnout latitude

SHEBO = sine of the burnout heading angle. If SHEBO is positive, i.e., the burnout heading between 0 and 180 degrees, PLAIN will be between 0 and 90 degrees, or a prograde orbit. If SHEBO is negative, i.e., the burnout heading between 180 and 360 degrees, then PLAIN will be between 90 and 180 degrees, or a retrograde orbit. The term CLABO will always be positive.

ACOS = arc cosine

### True Anomaly at Burnout (VEE)

The angle between the perigee and the point in the orbit at burnout is the initial true anomaly of the ellipse and is given by the following relation. [2]

$$\text{VEE} = \text{true anomaly} = \tan^{-1} \left[ \frac{\frac{(\text{HBO}) (\text{VBO})^2}{\text{GMU}} \sin (\text{GBO}) \cos (\text{GBO})}{\frac{(\text{ALT}) (\text{VBO})^2}{\text{GMU}} \cos^2 (\text{GBO}) - 1} \right] \quad (6)$$

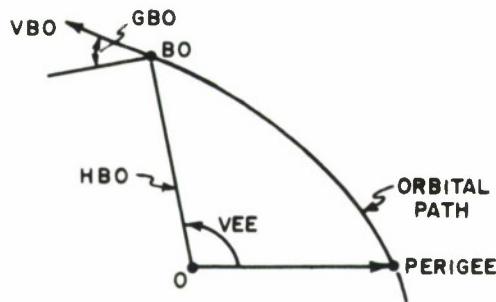
where GMU is the gravitational mass unit or constant K in Reference [2]. The magnitude of the true anomaly depends on the signs of the numerator and denominator, both of which can be plus or minus independently of the other. In the numerator, the burnout flight path angle GBO can be plus or minus relative to the local horizon. The term reflecting this sign is  $\sin (\text{GBO})$  which can be either plus or minus. In the denominator, the first term can be greater or less than one. The sign is primarily determined by the burnout velocity.

The simulation logic must be arranged to provide the proper value of the true anomaly for different combinations of signs of the numerator and denominator at burnout. A geometric interpretation of these different combinations is given in Fig. 5. Detailed Fortran logic of this part of the program is given on lines T38 through T41 in Appendix II.

### Eccentricity (ECC)

The eccentricity of the ellipse is given by the equation: [2]

$$\text{ECC} = \sqrt{\left[ \frac{(\text{HBO}) (\text{VBO})^2}{\text{GMU}} - 1 \right] \cos^2 (\text{GBO}) + \sin^2 (\text{GBO})} \quad (7)$$

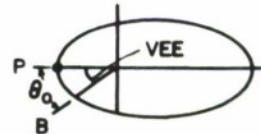


$$A = \frac{(HBO)(VBO)^2}{GMU}$$

CASE 1     $A \cos^2 GBO - I > 0$   
 $\sin GBO > 0$

$$\theta_0 = \tan^{-1} \frac{(+)}{(+)}$$

$$VEE = \theta_0$$

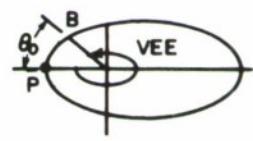


CASE 2     $A \cos^2 GBO - I > 0$   
 $\sin GBO < 0$

$$\theta_0 = \tan^{-1} \frac{(-)}{(+)}$$

$$= -\tan^{-1} \frac{(+)}{(-)}$$

$$VEE = 360 - |\theta_0|$$



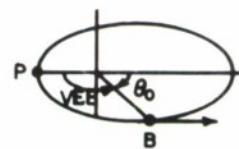
CASE 3     $A \cos^2 GBO - I < 0$   
 $\sin GBO > 0$

$$\theta_0 = \tan^{-1} \frac{(+)}{(-)}$$

$$= -\tan^{-1} \frac{(+)}{(-)}$$

$$VEE = 180 - |\theta_0|$$

$$= 180 + \theta_0$$



CASE 4     $A \cos^2 GBO - I < 0$   
 $\sin GBO < 0$

$$\theta_0 = \tan^{-1} \frac{(-)}{(-)}$$

$$= \tan^{-1} \frac{(+)}{(+)}$$

$$VEE = 180 + |\theta_0|$$

$$= 180 + \theta_0$$

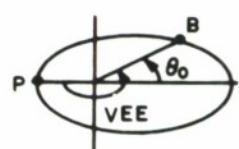


Fig. 5      Quadrant Determination of True Anomaly at Burnout

where

HBO = altitude in nautical miles from earth center to burnout altitude

VBO = burnout velocity in knots

GMU = gravitational mass unit =  $gR_e^2 = 62,750.21 \text{ n.m.}^3 \text{ per sec.}^2$

GBO = burnout flight path angle

### Semi-latus Rectum (P)

The distance from the focal point of the ellipse to a point on the elliptical orbit where the true anomaly is equal to 90 degrees is called the semi-latus rectum or the parameter of the orbit (the line FB in Fig. 4.) It is given by the polar equation for a conic. [3]

$$P = HBO (1 + ECC * \cos VEE) \quad (8)$$

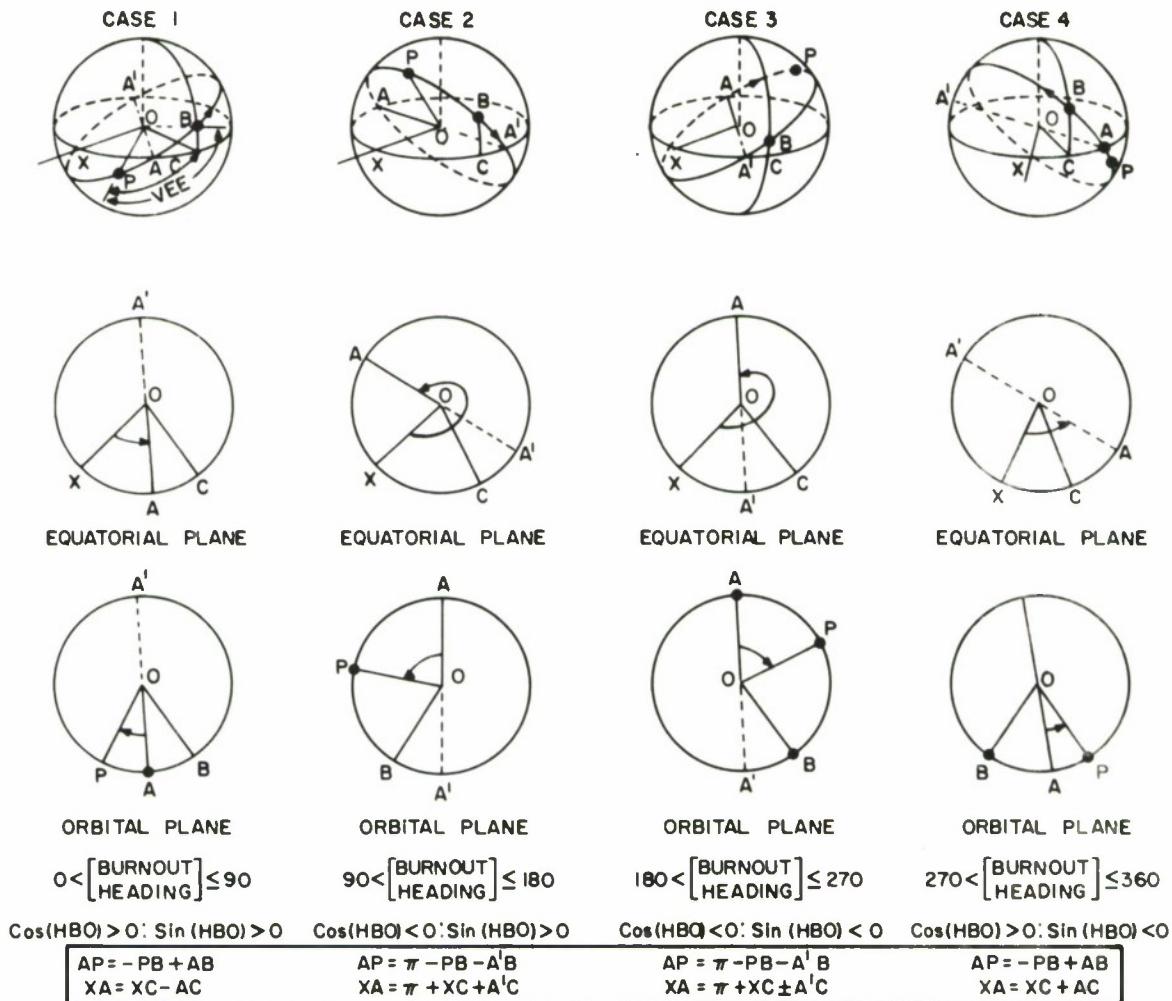
where these quantities have all been defined previously.

### Argument of Perigee at Burnout Relative to the Ascending Node (OMEGA)

The argument of perigee, OMEGA in Fortran format, is generated from the true anomaly (VEE) and the arc AB of the spherical triangle ABC of Fig. 4. The relations which exist between OMEGA, true anomaly, and arc AB depend upon the burnout heading and latitude.

In Fig. 6, the possible permutations of these two burnout parameters and the corresponding expressions for the argument of perigee OMEGA are given. For any given burnout heading quadrant, it should be noted that changing from a northern to southern hemisphere burnout location merely changes the sign of the arc AB in the expression for OMEGA (AP). Recognition of this characteristic simplifies the Fortran program logic.

(a) NORTHERN HEMISPHERE  
( $\sin(\text{LAT}) > 0$ ;  $\cos(\text{LAT}) > 0$ )



WHERE : PB = TRUE ANOMOLY (VEE)

AP = ARGUMENT OF PERIGEE RELATIVE TO ASCENDING NODE (OMEGA)

XA = LONGITUDE OF ASCENDING NODE (ASNODE)

XC = LONGITUDE OF BURNOUT (BOLONG)

AB = ARGUMENT OF BURNOUT RELATIVE TO ASCENDING NODE IN ORBIT PLANE

AC = ARGUMENT OF BURNOUT RELATIVE TO ASCENDING NODE IN EQUATORIAL PLANE

Fig. 6 Quadrant Resolution of Argument of Perigee and Ascending Node

(b) SOUTHERN HEMISPHERE  
( $\sin(\text{LAT}) < 0 : \cos(\text{LAT}) > 0$ )

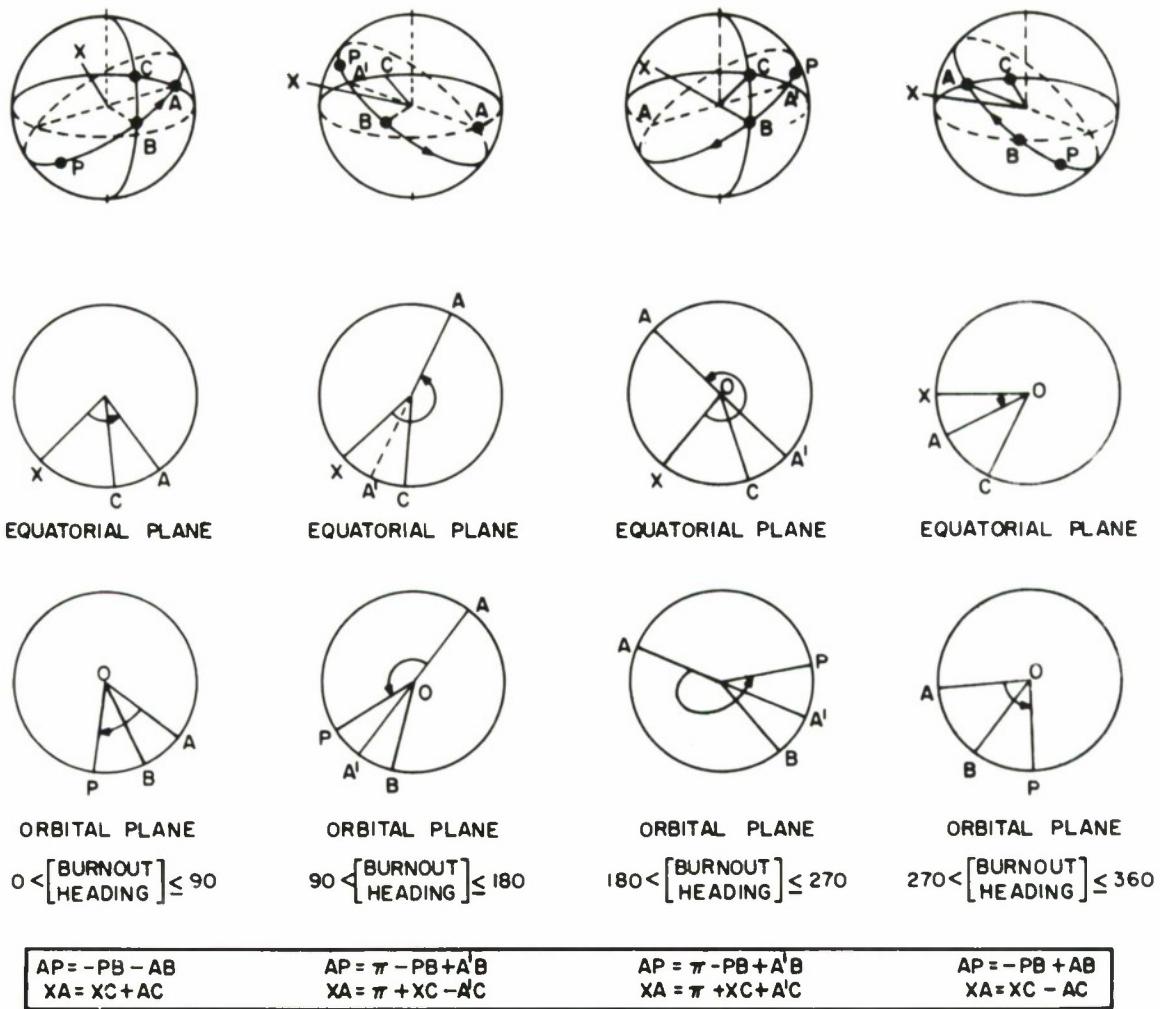


Fig. 6 (Continued) Quadrant Resolution of Argument of Perigee and Ascending Node

The trigonometrical relations for the evaluation of AB follows directly from the sine law. Referring to Fig. 4,  $\sin C = \sin 90^\circ = 1$

$$AB = ARC \sin \left[ \frac{\sin BC}{\sin (PLAIN)} \right] = ARC \sin \left[ \frac{\sin (\text{latitude})}{\sin (\text{inclination})} \right] \quad (9)$$

or in Fortran notation

$$AB = \text{ASIN } (\text{SLAIN})^* \quad (10)$$

where  $\text{SLAIN} = \text{SLABO}/\text{SINL}$

The Fortran listing of the branching logic for this portion of the program is contained on lines T54 through T61.1 in Appendix II.

#### Longitude of the Ascending Node (ASNODE)

The longitude of the ascending node is called ASNODE in the Fortran program. Its value also depends on the burnout heading and latitude. Prograde and retrograde launchings will generate ascending nodes differing by 180 degrees due to the definition of the ascending node. The relations for the longitude of the ascending node are given in Fig. 6 for the possible burnout latitude and heading permutations. Similar to the argument of perigee, changing from a northern to a southern burnout location at a given heading merely changes the sign of the arc AC in the expression for the ASNODE (arc XA, Fig. 6).

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\* The ASIN subroutine treats a negative angle as minus the arc sine of the magnitude of the angle. This is consistent with the branching logic, since AB will be positive for the Northern Hemisphere and negative for the Southern Hemisphere.

The trigonometrical expression for arc AC in terms of known orbital data can again be derived directly from the basic spherical triangle in Fig. 4 and the sine law.

$$\sin C = 1 \therefore \sin AC = \sin AB \sin B \quad (11)$$

Substituting for  $\sin AB$  from Eq. (9)

$$\sin AC = \frac{\sin BC \sin B}{\sin (\text{PLAIN})} \quad (12)$$

but B is the burnout heading; thus, in Fortran notation,

$$\begin{aligned} AC &= \text{ASIN (SLABO * SHEBO / SINL)} \\ \text{or } AC &= \text{ASIN (SLAIN * SHEBO)} \end{aligned} \quad (13)$$

where SHEBO is the sine of the burnout heading angle.

The corresponding Fortran listing of the branching logic is contained on lines T54 through T61.1 in Appendix II.

#### CALCULATION OF TIME-VARYING ORBITAL PARAMETERS

The fixed parameters given above establish the initial position of the vehicle relative to the earth. From these six initial conditions and the differential equation of motion for the variation of the true anomaly with time, the position of the vehicle relative to the moving earth can be determined. The variation of the true anomaly is obtained by a numerical integration of the equation of motion of the orbiting vehicle. This calculation is performed in subroutine RUNG whose calling is controlled by subroutine TRAC. Subroutine

RUNG then outputs the true anomaly to subroutine TRAC where the calculation of the following three parameters defining the vehicle position relative to a moving earth are made.

RLAT = Latitude of vehicle on moving earth.

RLONG = Longitude of vehicle on moving earth.

ALT = Altitude of vehicle above spherical earth.

The calculation of the time-varying orbital parameters begins with statement number 700 of the TRAC listing in Appendix II.

#### Latitude of Vehicle on Moving Earth (RLAT)

The rotation of a spherical earth has no net effect on the latitude of a vehicle relative to the earth. An expression relating the latitude at time  $t_i$  and the parameters previously given can be readily obtained from the spherical triangle ABC in Fig. 4.

From the law of sines

$$\sin (RLAT) = \sin BC = \sin (PLAIN) \sin AB \quad (14)$$

$$AB = VEE + OMEGA \quad (15)$$

Substituting into Eq. (14) and expanding the sine

$$\therefore \sin (RLAT) = \sin (PLAIN) [\sin (VEE) \cos (OMEGA) + \cos (VEE) \sin OMEGA] \quad (16)$$

In Fortran listing the following notation is used.

$$\sin(RLAT) = SLAR; \sin(VEE) = SVEE; \sin(OMEGA) = SOM$$

$$\sin(PLAIN) = SINL; \cos(VEE) = CVEE; \cos(OMEGA) = COM$$

$$C_3 = SOM * SINL$$

$$C_4 = COM * SINL$$

where VEE is now the true anomaly at time  $t_i$

$$\therefore SLAR = C_4 * SVEE + C_3 * CVEE \quad (17)$$

$$RLAT = ASIN(SLAR) * CONVI \quad (18)$$

where CONVI is a conversion factor from radians to degrees. The geometric relationships for OMEGA and VEE were given in Fig. 6.

#### Longitude of Vehicle on Moving Earth (RLONG)

The earth's rotation will affect the longitude of the vehicle to the earth, since the reference from which the longitude is measured is fixed in the earth. Let WE be the angular velocity of the earth; then at time  $t_i$ , the longitude is given by the expression:

$$RLONG(t_i) = CONGR = ASNODE - WE(t_i) \pm AC \quad (19)$$

where from the cosine law AC is given by the following expression:

$$AC = \cos^{-1} \frac{\cos(AB)}{\cos(BC)} = \cos^{-1} \frac{\cos(OMEGA + VEE(t_i))}{\cos(LAT)} \quad (20)$$

Thus, in terms of previously defined Fortran notation

$$\text{CONGR} = \text{ASNODE} \pm \text{ACOS}(\text{COM} * \text{CVEE} - \text{SOM} * \text{SVEE}) / (\text{CLAR}) - \text{WE} * \text{TIME} \quad (21)$$

and

$$\text{RLONG} = \text{CONGR} * \text{CONVI} \quad (22)$$

A plus sign with the AC term in the expression CONGR signifies a prograde orbit; a minus sign signifies a retrograde orbit. The burnout heading angle which determines whether the motion is prograde or retrograde can be used to control the branch logic. This logic is contained on lines T82 through T87 in Appendix II.

The quadrant of the term  $\cos(\Omega + VEE(t_i))$  in the above expression for AC is lost when this angle exceeds  $\pi$ . That is, the particular inverse cosine subroutine used in the program only gives angle values between 0 and  $\pi$ . This programming difficulty was circumvented by using the sine of the latitude (SLAR) to control the branching to either the plus or minus form of the above expression for CONGR. By definition of the ascending node, the angle  $(\Omega + VEE)$  is between  $\pi$  and  $2\pi$  in the Southern Hemisphere. The nature of these relationships are illustrated in Fig. 7.

#### Orbital Altitude (ALT)

For a spherical-earth model, the only time-varying parameter in the expression for orbital altitude is the true anomaly, or

$$ALT = P / (1.0 + \cos(VEE(t_i))) \quad (23)$$

where

P = the semi-latus rectum

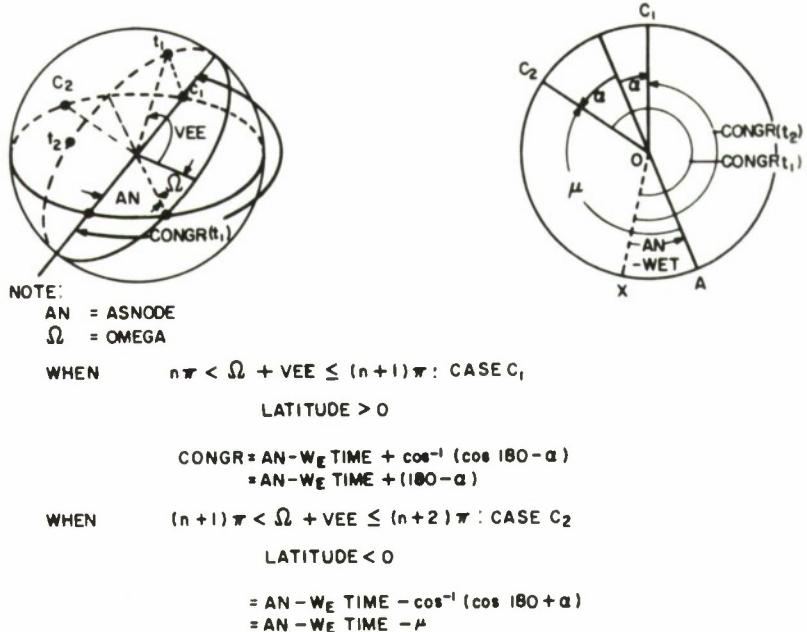


Fig. 7      Resolution of Longitude Ambiguity

Note that this is the same as Eq. (8) except that VEE is the true anomaly at time  $t_i$  rather than at burnout.

#### DIRECTION COSINES OF THE SPIN-STABILIZED ANTENNA (X<sub>I</sub>, Y<sub>I</sub>, Z<sub>I</sub>)

The calculation of the direction cosines of the spin-stabilized antenna axis relative to the X, Y, Z earth-centered coordinates are made in subprogram TRAC. This represents a slight deviation from the basic approach of having each subroutine perform a self-contained engineering function. These direction cosines are passed to subroutine CALC for computing the geometrical relationships between the orbital vehicle antenna axis and the ground sensors. These direction cosines involve the same functions of the burnout conditions as the orbital parameters at burnout; therefore, by making the calculation in TRAC,

the necessity of passing all these functions to CALC is eliminated. The derivation of these direction cosines is given in Appendix VII.

## CONTROL FUNCTIONS

In addition to the orbital calculations just described, TRAC subroutine functions as the primary controller of the over-all program. The control constants for the program are contained on input card #2 which is read into the program in this subroutine. As described in Section II, card #2 contains the sixteen control constants, NUM (1) through NUM (16). By proper choice of these constants it is possible to control the integration interval, printout intervals and the printout increments within the intervals, and the selection of the subroutine CALC and program time stop options. Assigned control constants not used in TRAC are passed on to the appropriate subroutines called by TRAC. Some of the storage for the control constants is not utilized; it was provided to accommodate future program modifications.

NUM (1) is the number of ground stations in the program and controls the loops which involve station parameter calculations. The program can accommodate up to 150 stations.

NUM (2) equals the number of integration increments in a printout increment. The program is set up so that the data need not be printed out after every integration interval. For example, if the integration increment, which is NUM (8), is set at 60 (in seconds), but data printout is desired only every 600 seconds, NUM (2) will be set equal to  $600/60 = 10$ . If data is desired at every integration increment, NUM (2) will be set equal to  $60/60 = 1$ .

NUM (3) is the length of the initial printout interval for the variable printout interval option. For example, it may be desirable to print out data every 10 seconds for the first 100 seconds of the trajectory. In that case, NUM (3)

will be set equal to 100. The variable printout interval option was incorporated into the program to accommodate the high-loft, suborbital type of trajectory. In this case, the initial and final parts of the trajectory are changing rapidly and a fine printout interval is desirable; whereas, in the middle part of the trajectory, a coarse printout is usually acceptable. The length of the initial phase is controlled by NUM (3). The altitude at the time in trajectory controlled by NUM (3) is then stored. When the trajectory reaches the same altitude on the return path, the program automatically reverts to NUM (3) control. Data printout then occurs at the same increment established for the initial phase, i.e., NUM (2) fixes the printout increment within the time interval established by NUM (3). The printout option will be bypassed when only orbital trajectories are considered. In that case, the trajectory is not divided into intervals. Printout occurs at the printout increment established by NUM (2) until the program time stop is reached.

NUM (4) is the number of integration increments in the intermediate printout interval. It does not affect the program if the variable printout interval option (controlled by NUM (6)) is not used.

NUM (5) controls the final program time. It only functions as a program control if NUM (6) is greater than one, i.e., when the variable printout option is by-passed. If NUM (6) is zero or negative, NUM (5) does not influence program control. The product NUM (5) x NUM (16), executed internally, controls the final program running time. For example, if NUM (16) = 1, NUM (5) represents the number of seconds to final orbit time. If NUM (16) = 60, NUM (5) represents the number of minutes to final orbit time.

NUM (6) controls the selection of the variable printout interval option. If the variable printout option is by-passed, the program will run until time equals

NUM (5) x NUM (16), and printout will occur at a constant increment throughout the program.

NUM (7) controls the calling of the subroutine CALC option. If it is greater than zero, the CALC option is selected.

NUM (8) is the integration interval in seconds.

NUM (9) controls the calling of the subroutine DEBE option. If it is greater than zero, the DEBE option is selected.

NUM (10) through NUM (15) are unassigned.

NUM (16) controls the final program time as explained under NUM (5). Essentially, NUM (16) permits longer program running times without upsetting the data card input format established for the NUM (I) series.

Subroutine TRAC does not output any data to the calling program. However, it does output orbit data if the CALC option is not selected. The input data required from the calling program are the burnout conditions as described in Section II. These data are normally supplied either by program MAIN or the powered flight trajectory simulation.



## SECTION IV

### GEOMETRICAL COVERAGE SIMULATION - SUBROUTINE CALC

The primary functions of subroutine CALC are the computation of the geometrical relations between the instantaneous position of the orbiting vehicle relative to the ground tracking stations, and the branch control of the DEBE subroutine option which calculates the space losses for various vehicle-ground antenna orientations. Figure 8 is a flow chart illustrating these functions.

#### STATION-VEHICLE GEOMETRY

In subroutine TRAC, the time-varying vehicle position relative to the equatorial plane and a reference meridian (Greenwich) was established. Latitude, longitude, and altitude above the earth were the three basic quantities which completely define this position. In subroutine CALC this basic position data is utilized to obtain the pertinent quantities relating the vehicle position to a ground tracking station. These quantities are the range (RHO) and range angle (PSI) from station to vehicle and the azimuth (AZD) and elevation (ELE) of the line of sight from station to vehicle.

From this data and a knowledge of the physical limitation of the tracking station equipment, tracking coverage of a vehicle by a given station for a given ground track can be obtained as a function of time. Stations below the minimum elevation angle are rejected by the program. If there are no local terrain limitations or inherent physical limitations in the equipment, the minimum elevation angle (ELEM) is defined as the angle between the local tangent plane and the line of sight tangent to the earth's surface. For a station located at sea level, the minimum elevation is zero. However, a minimum angle of

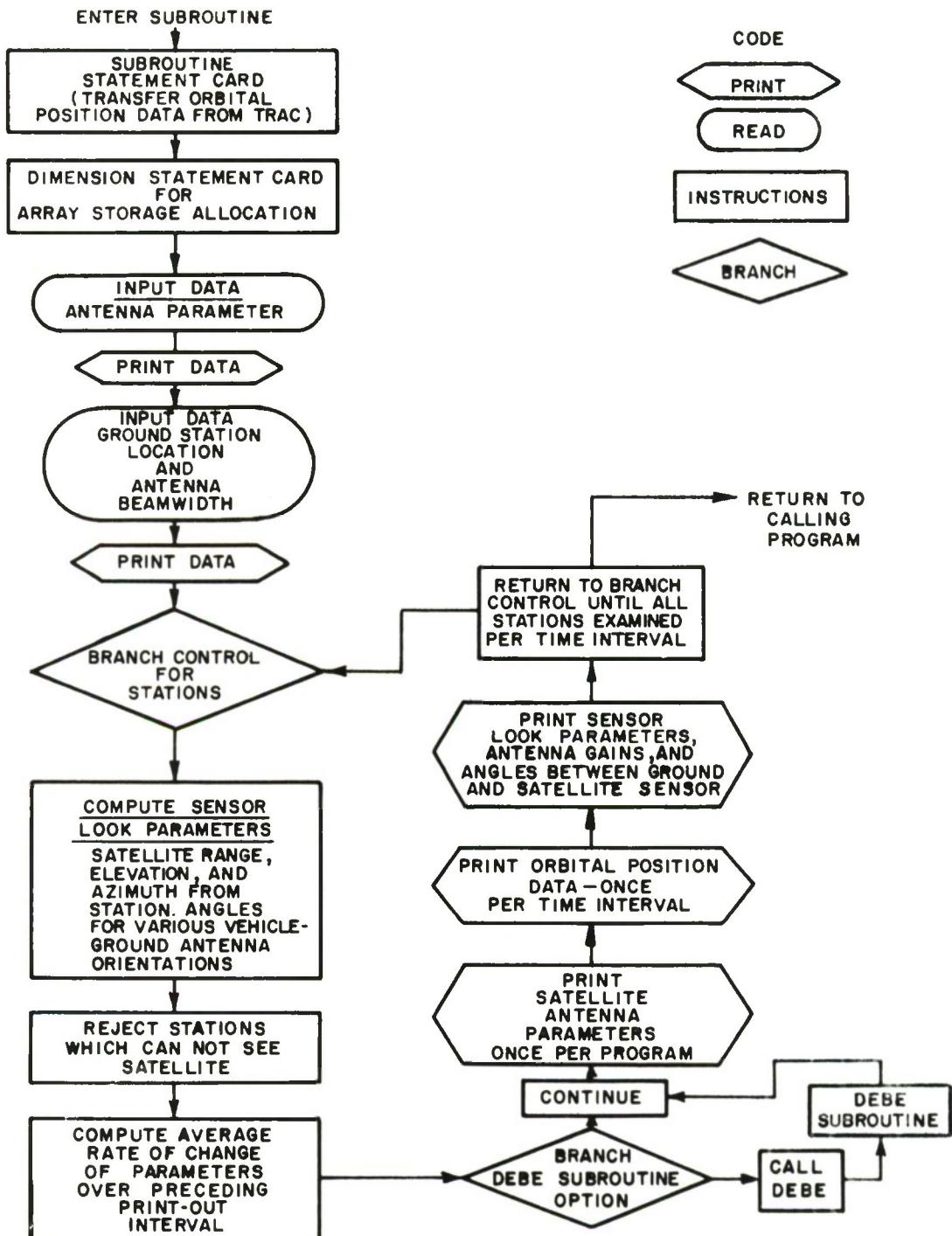


Fig. 8 Flow Diagram--Subroutine CALC

5 degrees is often used to allow for local obstructions in the case of the optical line of sight and atmospheric effects in the case of the electromagnetic line of sight. In any event, this angle is selected by the programmer to suit the application and is inserted into the program along with other station data (see Table 5). The nomenclature of the parameters defining the position of the vehicle relative to the earth and an arbitrary tracking station is summarized in Fig. 9.

#### Line-of-Sight Range from Station to Vehicle (RHO)

The cosine law applied to the plane triangle OBS in Fig. 9 yields the range directly.

$$RHO = \sqrt{(ALT)^2 + (RE + ALTS)^2 - 2(ALT)(RE + ALTS) \cos(PSI)} \quad (24)$$

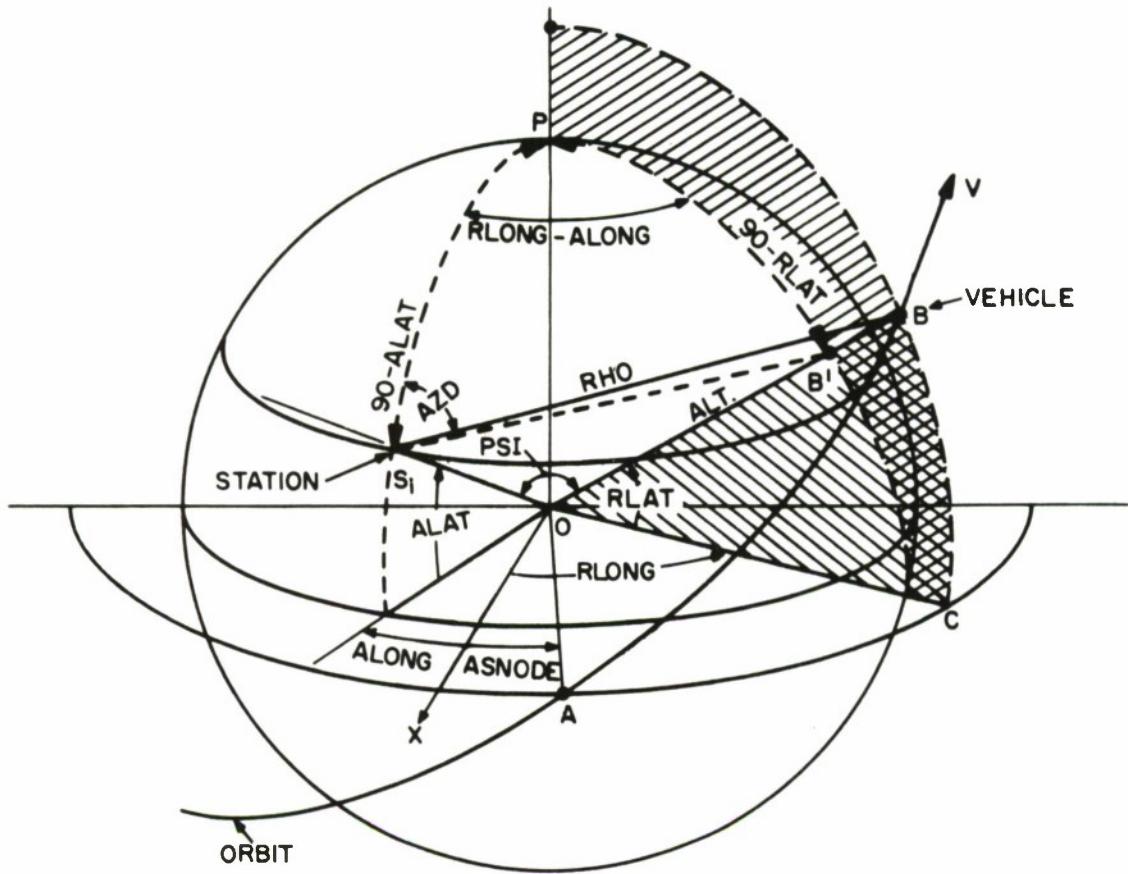
#### Range Angle (PSI)

The range angle is defined as the angle between the radius vectors from the center of the earth to the station and vehicle, respectively. It can vary between 0 and 180 degrees. Application of the cosine law to the spherical triangle PSB in Fig. 9 yields (actually, only the cosine of PSI and not PSI itself is required for the simulation)

$$\begin{aligned} \cos(PSI) &= \cos(90 - ALAT) \cos(90 - RLAT) + \sin(90 - ALAT) \\ &\quad \times \sin(90 - RLAT) \cos(RLONG - ALONG) \end{aligned} \quad (25)$$

Substituting compliments and expanding the  $\cos(RLONG - ALONG)$  gives:

$$\begin{aligned} \cos(PSI) &= \sin(ALAT) \sin(RLAT) + \cos(ALAT) \cos(RLAT) [\cos(ALONG) \\ &\quad \cos(RLONG) - \sin(ALONG) \sin(RLONG)] \end{aligned} \quad (26)$$



$S_i$  = STATION LOCATION

$B$  = VEHICLE LOCATION

ALONG = LONGITUDE OF STATION

RLONG = LONGITUDE OF VEHICLE

ALAT = LATITUDE OF STATION

RLAT = LATITUDE OF VEHICLE

AZD = AZIMUTH OF VEHICLE FROM STATION MEASURED FROM NORTH CLOCKWISE

PSI = RANGE ANGLE OR ANGLE BETWEEN STATION AND VEHICLE MEASURED FROM EARTH CENTER

RHO = DISTANCE BETWEEN STATION AND VEHICLE

ALT = ALTITUDE FROM EARTH CENTER TO VEHICLE

ALTS = ALTITUDE FROM EARTH SURFACE TO SENSOR

RE = RADIUS OF THE EARTH

$OS_i$  = RE + ALTS

Fig. 9      Station-Vehicle Geometry

In Fortran notation this becomes

$$CPSI = SLA(L) * SLAR + CLA(L) * CLAR * (CLO(L) * CLOR + SLO(L) * SLOR) \quad (27)$$

where L is the station index.

#### Azimuth Angle (AZD)

The application of the cosine law to the spherical triangle PSB in Fig. 9 yields

$$\cos (AZR) = \frac{\sin (RLAT) - \cos (PSI) \sin (ALAT)}{\sin (PSI) \cos (ALAT)} \quad (28)$$

In Fortran  $\cos (AZR) = \text{PHI}$ , and  $\sqrt{1 - \cos^2 (\text{PSI})}$  is substituted for  $\sin (\text{PSI})$  giving

$$\text{PHI} = (\text{SLAR} - CPSI * SLA(L)) / CLA(L) * SQRT (1.0 - CPSI**2) \quad (29)$$

$$\therefore AZR = A \cos (\text{PHI})$$

$$\text{and } AZD = AZR * CONVI \quad (30)$$

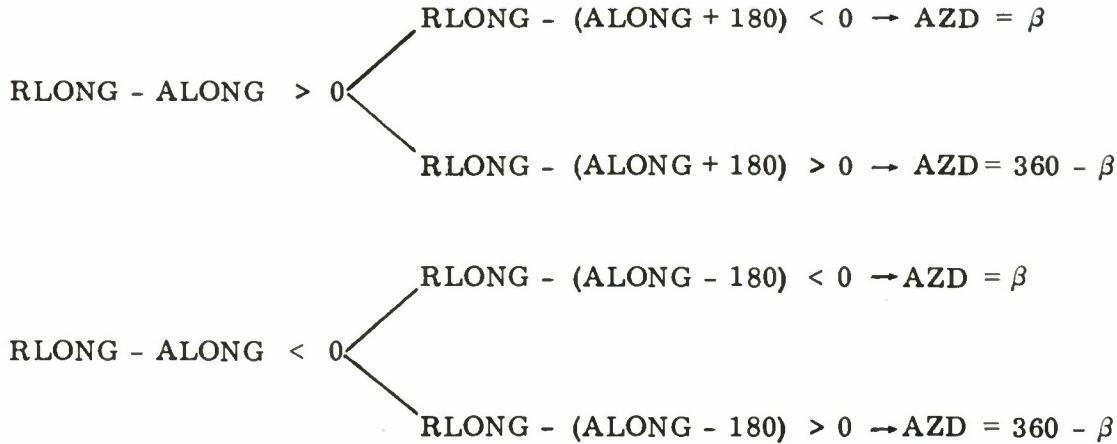
where CONVI is a conversion factor from radius to degrees.

Because the azimuth or heading angle between the tracking station and the vehicle in orbit is measured from north in a clockwise direction, a discontinuity in the heading angle can occur for certain orbital paths relative to the ground station. If the vehicle latitude, for example, is greater than the station latitude, and the orbital path remains north of the station, the measured azimuth track angle will vary from some value between + 90 and 0 degrees through 0 to some

value between 360 and 270 degrees. The appropriate branch logic control must be provided in the simulation to account for such orbital tracking situations.

The permutations in the heading angle which may occur for possible vehicle tracking station locations are illustrated in Fig. 10. Two types of orbital paths have to be given special consideration. In the first type, the orbit passes from the first to fourth quadrant in the heading angle but does not cross the discontinuity in longitude caused by measuring longitude east and west of the Greenwich meridian. In the second type the orbit passes from the first to the fourth quadrant in heading and also crosses the discontinuity in the longitude at  $\pm 180$  degrees.

As shown in Fig. 10, the proper value of the heading angle AZD can be obtained by branching on tests which compare the longitude of the station and vehicle. The following branching logic takes into account all possible permutations. Let  $\beta$  be the azimuth angle computed by Eq. 28, then if



The Fortran listing for this portion of the program is given on lines C16 through C23 in Appendix III.

a. VEHICLE LONGITUDE EAST OF GREENWICH  
 $(0 < R\text{LONG} \leq 180)$

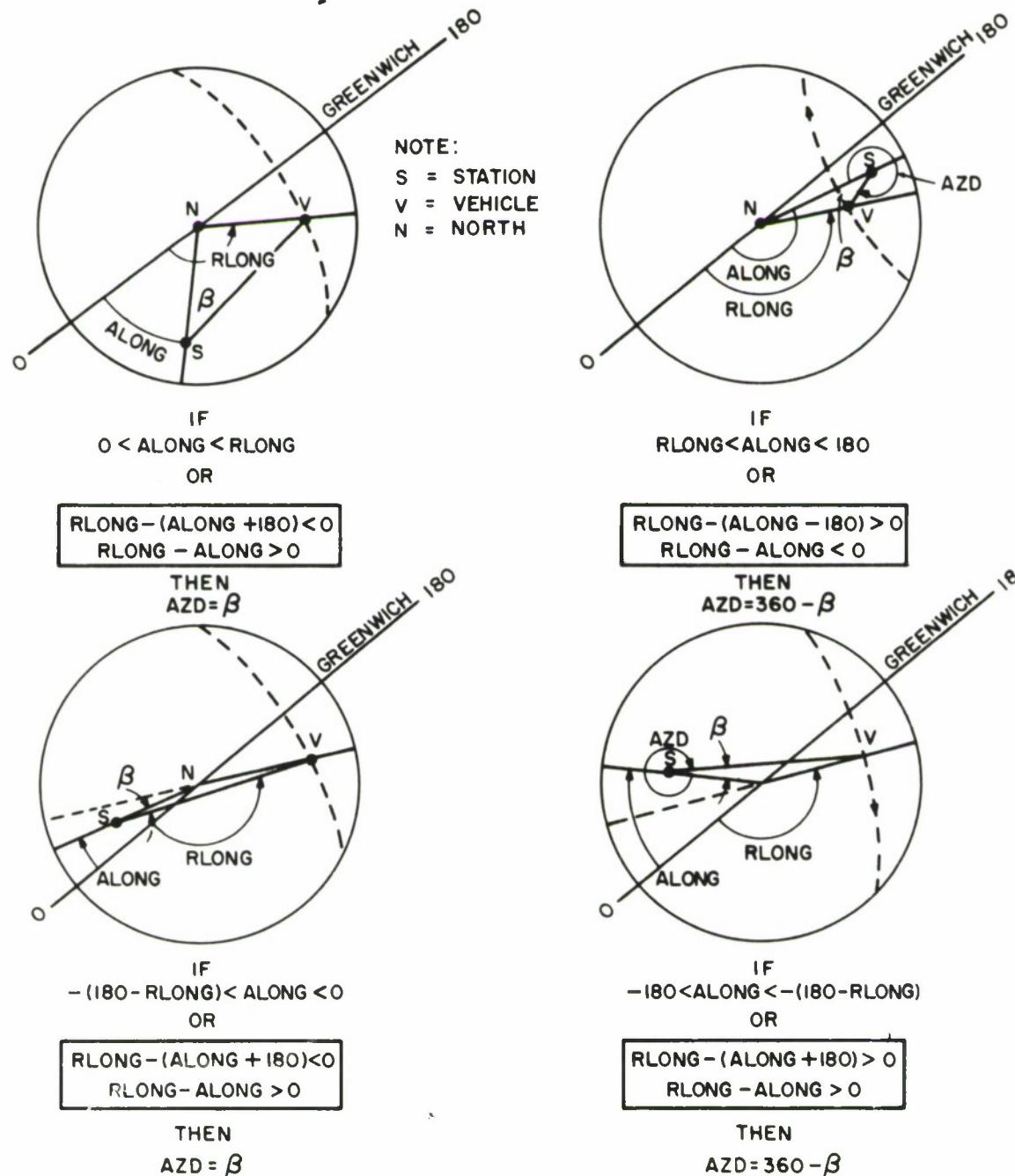


Fig. 10 Resolution of Heading Ambiguity

b. VEHICLE LONGITUDE WEST OF GREENWICH  
 $-180 < R\text{LONG} < 0$

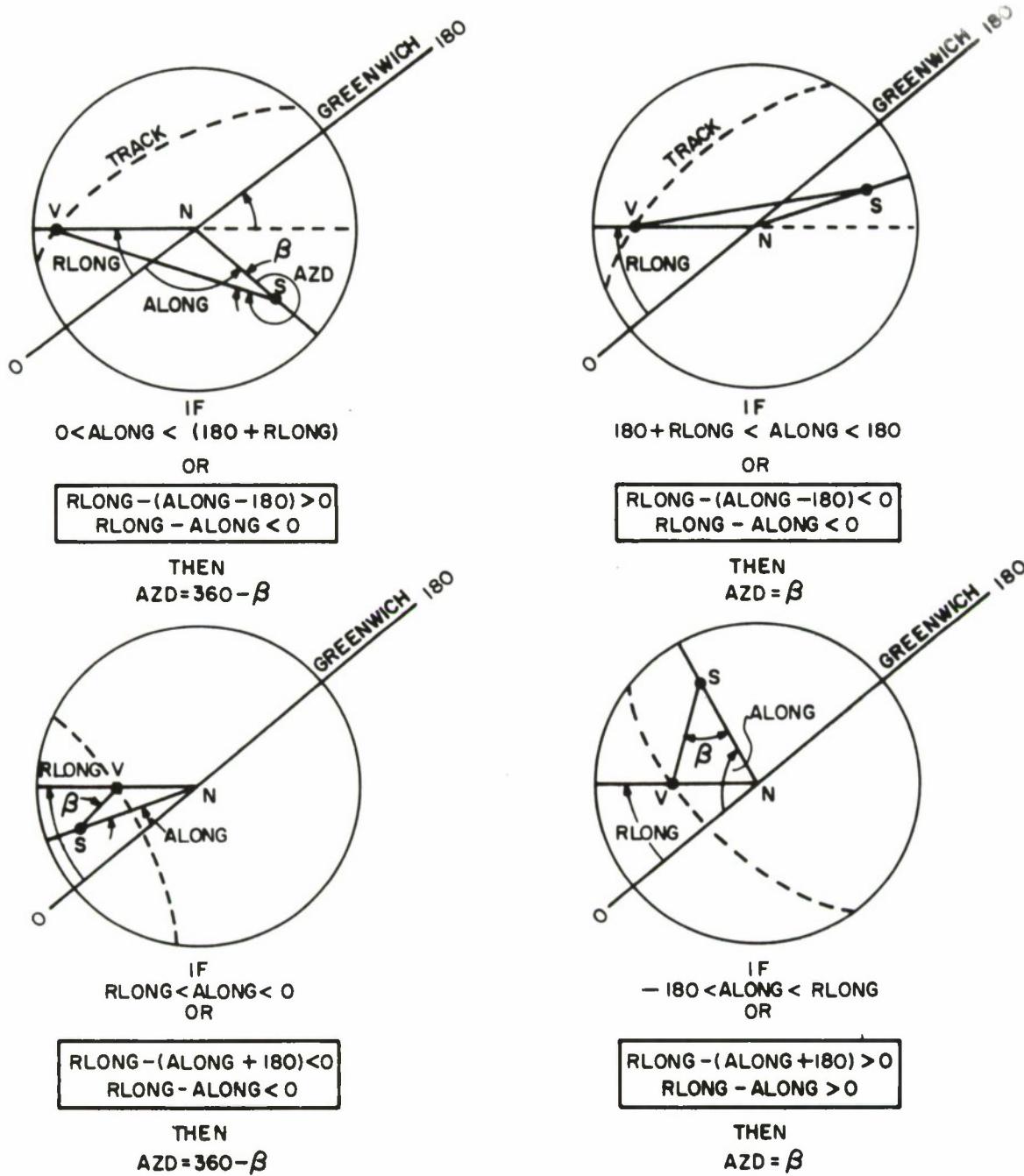


Fig. 10 (Continued) Resolution of Heading Ambiguity

### Elevation Angle (ELE)

The elevation angle can be found by applying the law of sines to the plane triangle SBS' in Fig. 11 (which is an expanded detail of the triangle OSB of Fig. 9).

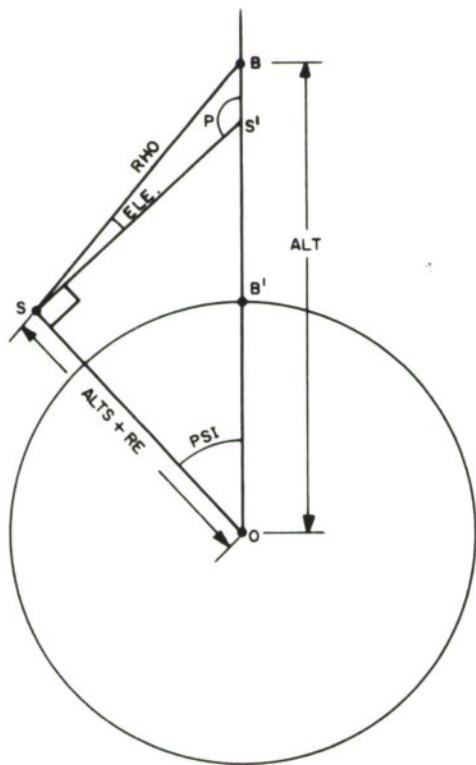


Fig. 11      Elevation Angle Geometry

$$\frac{\sin(ELE)}{BS'} = \frac{\sin \rho}{RHO} \quad (31)$$

$$\sin \rho = \sin(180 - (90 - \text{PSI})) = \cos \text{PSI} \quad (32)$$

$$BS' = ALT - \frac{(RE + ALTS)}{\cos \text{PSI}} \quad (33)$$

Substituting Eq. (32) and (33) into (31) and solving for sin (ELE) gives

$$\sin (\text{ELE}) = \frac{\text{ALT} \cos (\text{PSI}) - (\text{RE} + \text{ALTS})}{\text{RHO}} \quad (34)$$

In Fortran notation

$$\text{SELE} = [\text{ALT} * \text{CPSI} - (\text{RE} + \text{ALTS})] / \text{RHO} \quad (35)$$

$$\text{ELE} = \text{ASIN} (\text{SELE}) \quad (36)$$

#### ANTENNA ASPECT ANGLES (ANGLE, FNGL, ALPHA)

In addition to vehicle-station geometry, CALC subroutine calculates the following angles which define various vehicle-sensor antenna axes orientations.

ANGLE - the angle between the line of sight from station to vehicle and the axis of the spin-stabilized vehicle antenna.

FNGL - the angle between the line of sight from station to vehicle and the axis of an earth-center-pointing vehicle antenna.

ALPHA - the angle between the line of sight from station to vehicle and the fixed axis of the i-th station.

The cosine of each of these angles is found by taking the dot product of unit vectors along the line of sight and the respective antenna axes. The required direction cosines which define these unit vectors are derived in Appendix VII. The above angles are given by the inverse cosine of Eqs. (73), (74), and (75) as follows.

$$\text{ANGLE} = \text{ACOS } (\text{XI} * \text{XRHO} + \text{YI} * \text{YRHO} + \text{ZI} * \text{ZRHO}) \quad (37)$$

$$\text{FNGLLE} = \text{ACOS } (\text{XF} * \text{XRHO} + \text{YF} * \text{YRHO} + \text{ZF} * \text{ZRHO}) \quad (38)$$

$$\text{ALPHA} = \text{ACOS } (\text{XAN(L)} * \text{XRHO} + \text{YAN(L)} * \text{YRHO} + \text{ZAN(L)} * \text{ZRHO}) \quad (39)$$

where

$\text{XI}, \text{YI}, \text{ZI}$  = the direction cosines of the spin-stabilized antenna

$\text{XF}, \text{YF}, \text{ZF}$  = the direction cosines of the spaceborne antenna pointing toward the center of the earth

$\text{XAN(L)}, \text{YAN(L)}, \text{ZAN(L)}$  = the direction cosines of the fixed antenna axis of the L-th ground station (or sensor)

#### RATES OF CHANGE

It is often desirable to know how fast a variable is changing. Therefore, the time rates of change of RHO, AZD, ELE, ANGLE, FNGLLE, and ALPHA are computed and printed out. In this program a simple linear difference between computation points is used. It is planned to incorporate expressions for the instantaneous rates of change of these quantities into the program in the very near future.



## SECTION V

### ELECTROMAGNETIC COVERAGE SIMULATION - SUBROUTINE DEBE

All antenna gain and space-loss calculations are made in one subroutine called DEBE. The use of this subroutine is optional and is controlled by the fixed point number, NUM(9), on the control parameter input card. In its present form, the subroutine calculates the total losses for eight different vehicle-ground station antenna configurations. However, by consolidating all of the antenna calculations in one subroutine, program flexibility is achieved, and other antenna orientations and performances can be studied by simply inserting the appropriate DEBE subroutine. A functional flow diagram of the required calculations is shown in Fig. 12.

The total losses are made up of the space loss (SLOS) and the gains (losses) of the various antennas. Free-space conditions are assumed; that is, interference and diffraction phenomenon due to meterological conditions and other factors are neglected. The various antenna combinations are summarized in Table 11, and the associated geometry is shown in Fig. 13.

#### SPACE LOSS (SLOS)

The total signal power received is given by the standard one-way, free-space propagation equation for the ratio of power received to power transmitted. [4, 5]

$$\frac{P_r}{P_t} = \left( \frac{\lambda}{4\pi R} \right)^2 g_r g_t \quad (40)$$

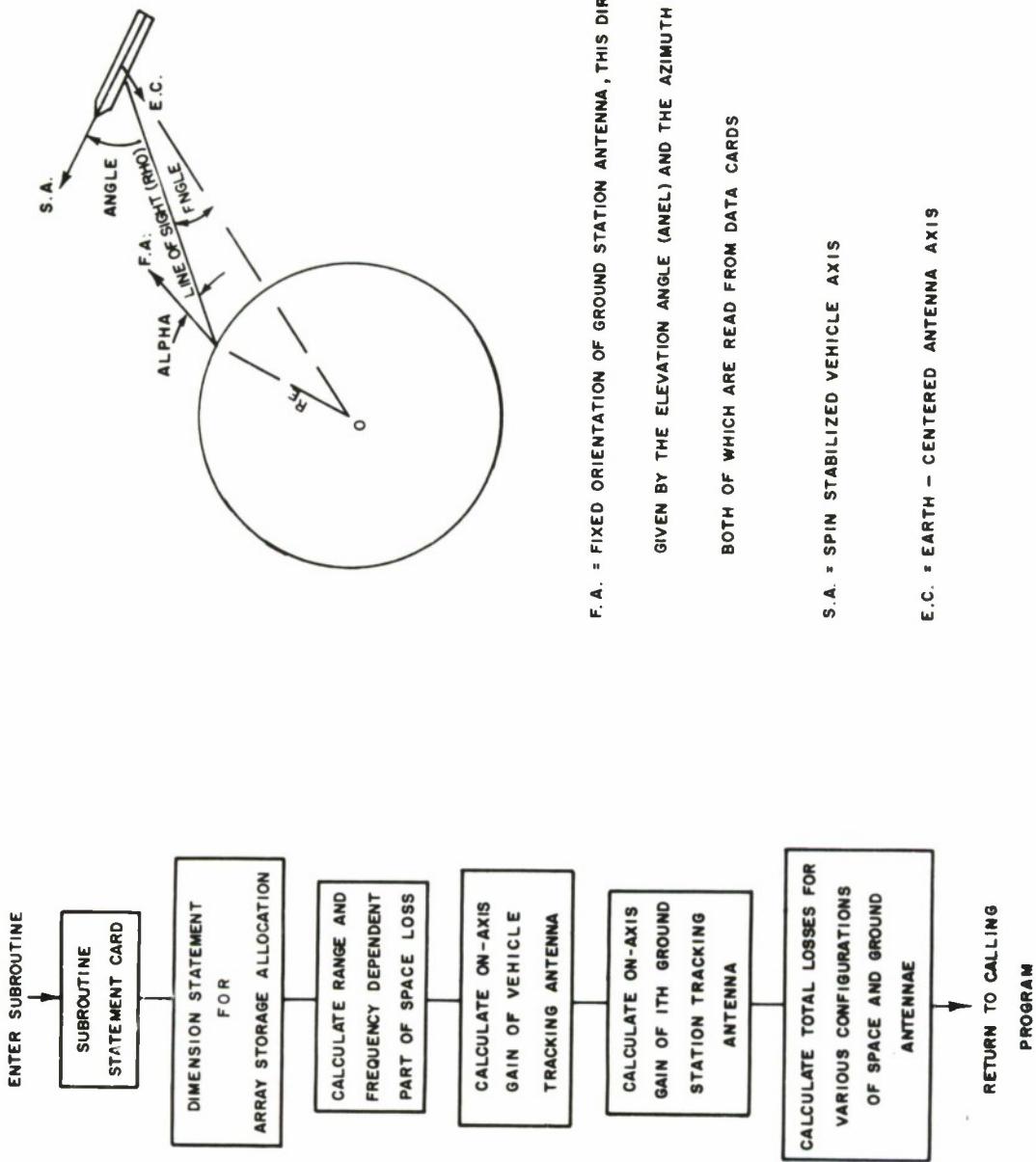


Fig. 12 Functional Flow Diagram—Subroutine DEBE

Fig. 13 Geometry for Space-Loss Calculations

where

$P_r$ ,  $P_t$  = received power and transmitted power, respectively.

$\lambda$  = wavelength, in the same units as R.

$g_r$ ,  $g_t$  = power gains of the receiving and transmitting antennas,  
respectively.

R = line of sight range.

Table 11  
Antenna Combinations

Associated Total Loss*	Vehicle Antenna		Ground Antenna	
	Direction (See Fig. 13)	Gain	Direction (See Fig. 13)	Gain
DB1	S. A.	GANGLE	F. A.	GALPHA
DB2	S. A.	GANGLE	RHO	DBA (L)
DB3	E. C.	GFNGLE	F. A.	GALPHA
DB4	E. C.	GFNGLE	RHO	DBA (L)
DB5	RHO	DBG	F. A.	GALPHA
DB6	RHO	DBG	RHO	DBA (L)
DB7	S. A.	GANGLE	Isotropic	0
DB8	E. C.	GFNGLE	Isotropic	0

\*Total loss equal space loss plus the gains of the antenna pair, i. e.,

$$DB1 = SLOS + GANGLE + GALPHA.$$

For purposes of computational flexibility the total loss as given by Eq. (40) is computed term-by-term in decibels ( $10 \log \frac{P_r}{P_t}$ ) and added to find the total loss. The total loss for unity antenna gains is designated as the space loss (SLOS) in the simulation, thus

$$SLOS = \left( \frac{\lambda}{4\pi R} \right)^2 = \left( \frac{c}{FREQ} \frac{1}{4\pi RHO} \right)^2 \quad (41)$$

where the wave length ( $\lambda$ ) is related to the system frequency (FREQ) by the speed of light (c) in vacuo. If the range (RHO) is given in nautical miles and FREQ in megacycles, then the speed of light which is  $299,776 \times 10^6$  meters/sec. must be expressed in nautical miles per second times  $10^{-6}$  or

$$c = \frac{(299.776)(3.28083)}{6076.115} \left( \frac{\text{n. m.}}{\text{sec.}} \times 10^{-6} \right)$$

In the simulation, space loss is computed as the sum of two components, the frequency-dependent space loss (FDSL), and the range-dependent space loss (RDSL) expressed in decibels. Thus,

$$SLOS = FDSL + RDSL$$

where

$$FDSL = 20. * \text{ALOG10} [ (299.776 * 3.28083 / 6076.115) / FREQ ] \quad (42)$$

and

$$RDSL = -20. * \text{ ALOG10} (4. * 3.1415927 * RHO) \quad (43)$$

## ANTENNA GAINS

### On-axis Gains (DBG, DBA)

For antennas which radiate (or receive) a well-defined beam, the fundamental relation which connects the on-axis or maximum gain  $G_o$ , the area of the aperture A, and the wavelength is:

$$G_o = f \frac{4\pi A}{\lambda^2} \quad (44)$$

The dimensionless factor  $f$  is equal to 1 if the excitation is uniform to phase and intensity over the whole aperture; in actual antennas,  $f$  is often as large as 0.8 and is rarely less than 0.5. The connection between gain and beamwidth is as follows. Using an aperture of dimensions  $d$  in both directions, a beam may be formed whose angular width\*, determined by diffraction, is very nearly

$$BW = \frac{\lambda}{d} \text{ radians} \quad (45)$$

If the aperture is square  $A = d^2$ , thus substituting for  $\lambda$  and  $A$  in Eq. (44), the gain becomes

$$G_o = f \frac{4\pi}{(BW)^2} \quad (46)$$

---

\*The beamwidth is the angular interval between two directions for which the gain is half of the maximum gain.

If the aperture is circular  $A = \frac{\pi}{4} d^2$ , the gain would be

$$G_o = f \left( \frac{\pi}{BW} \right)^2 \quad (47)$$

In DEBE subroutine, Eq. (46) is used for the maximum gain of the transmitting and receiving antennas. The transmitting antenna is considered to be on the space vehicle, and since it is a transmitting antenna close to the source of illumination, a value of 0.8 was selected for the factor  $f$ . This reasoning is also consistent with the type of antenna likely to be used on the vehicle, such as a helix or a dipole, which have higher efficiencies than a dish-type likely to be used on the ground. Thus, the maximum gain of the vehicle antenna is

$$DBG = \frac{0.8 (4\pi)}{(BW)^2} = 0.8 \left( \frac{3.54491}{BW} \right)^2 \quad (48)$$

For the receiving antenna on the ground where the illumination is likely to be a less uniform factor,  $f = 0.6$  was selected. The ground-antenna maximum gain is, therefore,

$$DBA(L) = 0.6 \left( \frac{3.54491}{BW(L)} \right)^2 \quad (49)$$

where in both instances  $BW$  is the beamwidth between half-power points in radians. Of course, for the isotropic-ground-antenna case, the gain is unity in all directions or zero db.

### Off-axis Gains (GANGLE, GFNGLE, GALPHA)

Cases are considered in which the vehicle and ground antennas are not completely directional relative to each other. To handle these cases, it is necessary to simulate the antenna pattern. There are an unlimited number of possible patterns. However, one that is fairly representative of many antennas and also was appropriate to the original problem (see Section 2, General Description) is the uniform distribution, which, in terms of the power gain, is given by

$$G = G_o \left( \frac{\sin \left( \frac{\pi d}{\lambda} \alpha \right)}{\frac{\pi d \alpha}{\lambda}} \right)^2 \quad (50)$$

where  $\alpha$  is the angle off the axis. It can be any of the angles (ANGLE, FNGLE, ALPHA) shown in Fig. 13. The half-power beamwidth can be expressed as

$$BW = K \frac{\lambda}{d} \quad (51)$$

which, when substituted in Eq. (50), gives

$$G = G_o \left( \frac{\sin \left( \frac{K\pi \alpha}{BW} \right)}{\frac{K\pi \alpha}{BW}} \right)^2 \quad (52)$$

The constant,  $K$ , can be determined from the condition at the half-power point, i.e., when  $\alpha = \frac{1}{2} BW$ , since the factor multiplying  $G_o$  must be equal to  $\frac{1}{2}$ .

$$\left( \frac{\sin \frac{K\pi}{2}}{\frac{K\pi}{2}} \right)^2 = \frac{1}{2} \quad (53)$$

Equation (53) is satisfied by a value

$$K\pi = 2.78332 \quad (54)$$

Thus, the final expression becomes

$$G = G_o \left[ \frac{\sin \left( \frac{2.78332\alpha}{BW} \right)}{\frac{2.78332\alpha}{BW}} \right]^2 \quad (55)$$

In Fortran notation, the three antenna gains are

the spin-stabilized antenna gain

$$GANGLE = 10. * ALOG10 (CON2 * ARG2) \quad (56)$$

the fixed-ground antenna gain

$$GALPHA = 10. * ALOG10 (CON1 * ARG) \quad (57)$$

the earth-centered antenna gain

$$GFNGLE = 10. * ALOG10 (CON2 * ARG3) \quad (58)$$

where

```
CON1 = DBA (L) = (3.54491/BW (L) * CONV)) ** 2 * 0.6  
CON2 = DBG      = (3.54491/BW1) ** 2 * 0.8  
ARG  = (SIN (AN1)/AN1) ** 2  
ARG2 = (SIN (AN2)/AN2) ** 2  
ARG3 = (SIN (AN3/AN3) ** 2
```

where

```
AN1   = 2.78332 * ALPHA/BW (L)  
AN2   = 2.78332 * ANGLE/BWG  
AN3   = 2.78332 * FNGLE/BWG
```

CONV is a conversion factor from degrees to radians



## SECTION VI

### SOLUTION OF THE ORBITAL EQUATION - SUBROUTINE RUNG

Subroutine RUNG performs the numerical integration of the Keplerian equation for the rate of change of the true anomaly of an elliptical orbit. The equation in Fortran notation is

$$\frac{d(\text{VEE})}{dt} = \frac{\text{GMU}}{p^{3/2}} [1. + \text{ECC} \times \cos(\text{VEE})]^2 = F(\text{VEE}) \quad (59)$$

where

VEE = true anomaly

ECC = eccentricity

GMU = gravitational mass unit ( $\text{nm}^3 \text{ sec}^{-2}$ )

P = semi-latus rectum

For a given elliptical orbit about a spherical earth, the vehicle position in the orbit is completely defined by the true anomaly (VEE) which is the angular position measured from the line of perigee. Hence, the numerical integration procedure which calculates the true anomaly establishes the position of the vehicle in a given elliptical orbit at the next time increment.

The input data to the subroutine is the true anomaly at the present instant of time (VEE(K) and the orbital parameters, K, P, and ECC. The subroutine

then calculates the true anomaly VEE (K+1) at the next instant of time as determined by the integration interval which can be arbitrarily selected.

The subroutine integrates the true anomaly equation given above by the following fourth-order Runge-Kutta integration technique.

$$\text{VEE (K+1)} = \text{VEE (K)} + (\text{VEER (1)} + 2(\text{VEER (2)} + \text{VEER (3)}) + \text{VEER (4)})/6 \quad (60)$$

where

$\text{VEE (K)}$  = the value of VEE at the start of the time increment

$\text{VEE (K+1)}$  = the value of VEE at the end of the time increment

$\text{VEER (1)}$  =  $F(\text{VEE (K)}) * \text{TNUM}$

$\text{VEER (2)}$  =  $F(\text{VEE (K)} + \frac{\text{VEER (1)}}{2}) * \text{TNUM}$

$\text{VEER (3)}$  =  $F(\text{VEE (K)} + \frac{\text{VEER (2)}}{2}) * \text{TNUM}$

$\text{VEER (4)}$  =  $F(\text{VEE (K)} + \text{VEER (3)}) * \text{TNUM}$

$\text{TNUM}$  = integration interval

The calculated value of  $\text{VEE (K+1)}$  is passed to subroutine TRAC where the calculations involving true anomaly are made. After these calculations,  $\text{VEE (K+1)}$  is returned to this subroutine and becomes  $\text{VEE (K)}$  in the calculation of the true anomaly at the succeeding time interval. This cycling is repeated until the program stops.

The errors in the true anomaly calculated by this numerical-integration technique have not been analytically studied in detail. However, the equation

for the rate of change of the true anomaly is well behaved, and the integration technique is adequate for the engineering applications for which the program is intended. A functional flow diagram of the subroutine is given in Fig. 14.

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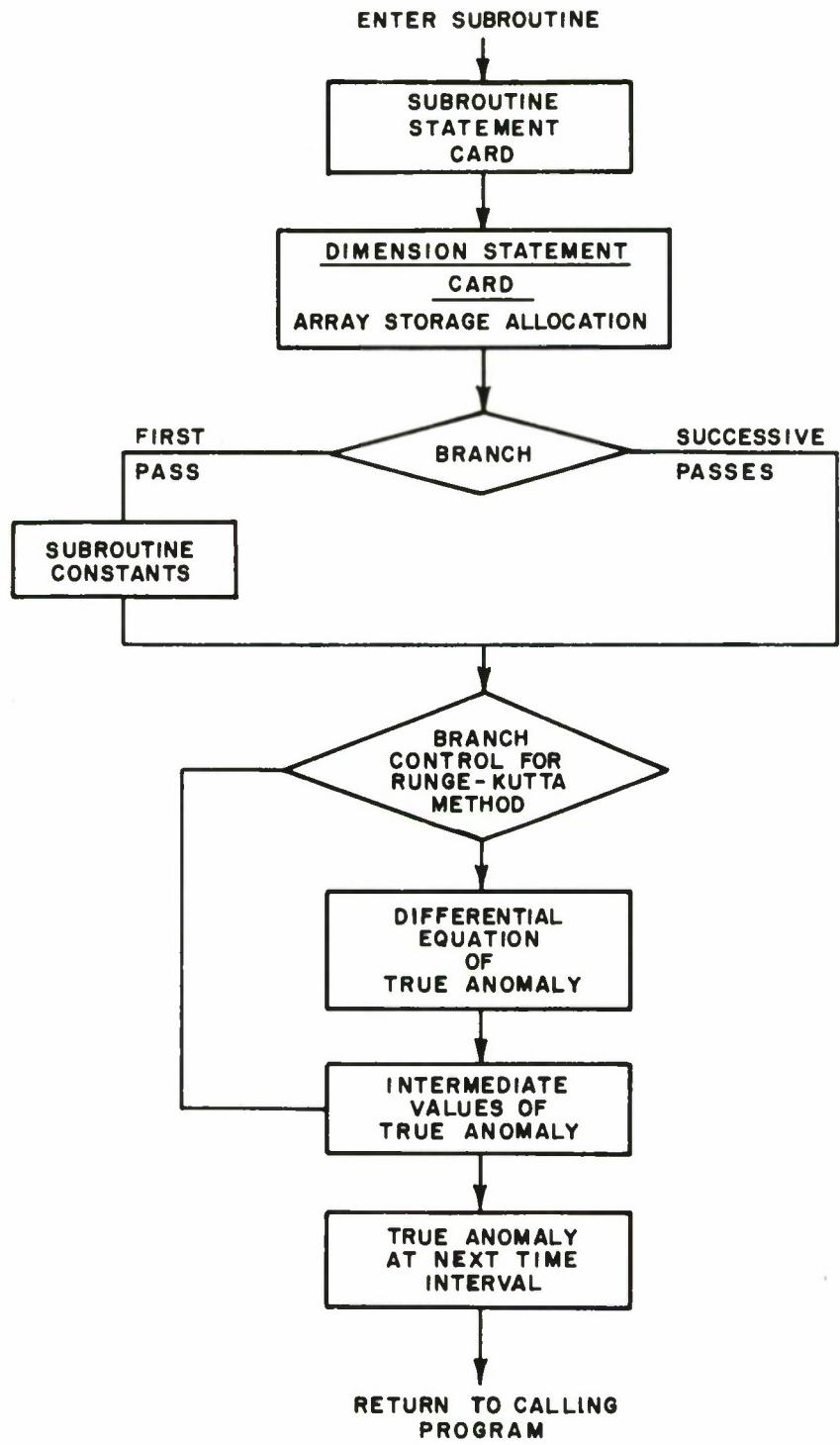


Fig. 14 Functional Flow Diagram--RUNGE-KUTTA Subroutine

APPENDIX I  
FORTRAN IV LISTING  
PROGRAM MAIN

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T      SUBTYPE,FORTRAN,LMAP
C      MODIFIED SPACE VEHICLE TRACKING AND SENSOR COVERAGE PROGRAM
C      BY J.STEVENS,L.WILKIE,D23,MITRE CORPORATION,BEDFORD,MASS
CMAIN
C      THIS PROGRAM IS MADE UP OF THE FOLLOWING SUBROUTINES-
C      TRAC -COMPUTES THE ORBITAL POSITION AND ATTITUDE OF THE VEHICLE.
C      RUNG -COMPUTES THE TRUE ANOMALY AS A FUNCTION OF TIME
C              BY A FOURTH ORDER RUNGE KUTTA NUMERICAL INTEGRATION METHOD
C      CALC -COMPUTES THE GEOMETRICAL RELATIONS BETWEEN THE VEHICLE
C              AND THE GROUND SENSORS.
C      DEBE -COMPUTES THE SPACE LOSSES AND THE GAINS OF VARIOUS VEHICLE-
C              GROUND STATION ANTENNA CONFIGURATIONS.
I READ 2,HBO,VBO,GBO,BOHEAD,BOLAT,BOLONI,BIG,BIH      M   1.
2 FORMAT(8F10.2)      M   2.
CALL TRAC (HBO,VBO,GBO,BOHEAD,BOLAT,BOLONI,BIG,BIH)  M   3.
IF(VBO)3,1,1      M   4.
3 RETURN      M   5.
END      M   6.
```



APPENDIX II  
FORTRAN IV LISTING  
SUBROUTINE TRAC

LIST OF SYMBOLS USED IN SUBROUTINE TRAC		
C	SYMBOL	UNITS
C	ALT	(N MI)
C	ALTI	(N MI)
C	ASNOOE	(RAD)
C	BETA	(DEG)
C	BIG	(OEG)
C	BIH	(OEG)
C	BOHEAO	(OEG)
C	BOLAT	(OEG)
C	BOLONG	(OEG)
C	C1	(NONE)
C	C2	(NONE)
C	ECC	(NONE)
C	GBO	(OEG)
C	HBO	(N MI)
C	NUM(1)	(NONE)
C	NUM(2)	
C	NUM(3)	(SEC)
C	NUM(4)	
C	NUM(5)	(SEC)
C	NUM(6)	(NONE)
C	NUM(7)	(NONE)
C	NUM(8)	
C	NUM(9)	(NONE)
C	NUM(16)	(NONE)
C	NUM2	(SEC)
C	NUM4	(SEC)
C	OMEGA	(RAO)
C	P	(N MI)
C	PLAIN	(OEG)
C	SNOOE	(DEG)
C	VBO	(FT/SEC)
C	VEE	(RAO)
C	VEE1	(OEG)
C	XI	
C	YI	(NONE)
C	ZI	
C	THIS SUBROUTINE CALCULATES VEHICLE POSITION IN GEOCENTRIC COORDINATES. THE INPUTS ARE THE BURNOUT CONDITIONS WHICH MAY BE OBTAINED EITHER FROM MAIN OR THE BOOST PROGRAMS (203 DEGREES OF FREEDOM).	T
C	THE SUBROUTINE USES TRAC TO MAKE IT COMPATIBLE WITH GENERAL CALLING STATEMENT	T
C	SUBROUTINE TRAC (HBO, VBO, GBO, BOHEAD, BOLAT, BOLONI, BIG, BIH )	T I.

```

DIMENSION NUM(16),VFF(5) T 2.
ROLONG=BOLONI T 2.01
IF(ISTART+1)191,193,191
191 ISTART=-1 T 2.21
C PHYSICAL CONSTANTS
PIE=3.1415927 T 2.3
HAPIE=1.5707963 T 2.4
CONV=0.01745329 T 3.
CONVI=57.29578 T 4.
CFEET=6076.115 T 5.
CNMI=0.0001645788 T 6.
RE=3438.149 T 7.
WE=2.0*PIE/B6400. T 8.
GMU=62750.21 T 9.
REA0_192,(NUM(L),L=1,16) T 9.5
192 FORMAT(16I5) T 9.6
NUM2=NUM(2)*NUM(8) T 10.
NUM4=NUM(4)*NUM(8) T 11.
NUM(5)=NUM(5)*NUM(16) T 11.01
PRINT 69 T 11.1
PRINT B7,(NUM(L),L=1,16) T 11.12
193 PRINT 72,HBO,VBO,GBO,BOHEAO,BOLAT,BOLONG,BIG,BIH T 12.1
HBO=HBO+RE T 12.2
VBO=VBO*CNMI T 13.
C CONVERT DEGREES TO RADIANS
GBO=CONV*GBO T 14
BOHEAR=CONV*BOHEAO T 15.
BOLAT=CONV*BOLAT T 16.
BOLONG=CONV*BOLONG T 17.
BIG=CONV*BIG T 18.
BIH=CONV*BIH T 19.
C COMPUTE SIN AND COS OF BURNOUT PARAMETERS
CHEBO=COS(BOHEAR) T 20.
SHEBO=SIN(BOHEAR) T 21.
COBIG=COS(BIG) T 22.
SOBIG=SIN(BIG) T 23.
COBIH=COS(BIH) T 24.
SOBIH=SIN(BIH) T 25.
COGBO=COS(GBO) T 26.
SOGBO=SIN(GBO) T 27.
CLABO=COS(BOLAT) T 28.
SLABO=SIN(BOLAT) T 29.
CLOB0=COS(BOLONG) T 30.
SLOB0=SIN(BOLONG) T 31.
C1=SOBIG*CLABO-COBIG*COBIH*SLABO T 32.
C2=COBIG*SOBIH T 33.
Z1=SOBIG*SLABO+COBIG*COBIH*CLABO T 33.1
C COMPUTE ORBITAL PARAMETERS, ECCENTRICITY, INCLINATION, TRUE ANOMALY
A=HBO*VBO**2/GMU T 34.
ECC=SORT(((A-1.0)*COGBO)**2+SOGBO**2) T 35.
PLAIN=ACOS(CLABO*SHEBO)*CONVI T 36.
D=A*COGBO**2-1.0 T 37.
VEE(1)=ATAN(A*SOGBO*COGBO/D) T 38.
IF(D)201,202,202 T 39.
201 VEE(1)=VEE(1)+PIE T 40.
202 CONTINUE T 41.
C COMPUTE OMEGA, ARGUMENT OF PERIGEE RELATIVE TO ASCENDING NODE
C COMPUTE ASNODE, LONGITUDE OF ASCENDING NODE MEASURED FROM THE
C GREENWICH MERIDIAN AND RELATIVE TO A NON-ROTATING EARTH
CINL=CLABO*SHEBO T 42.

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SINL=SORT(1.-CINL**2) T 43.
SLAIN=SLABO/SINL T 44.
AB=ASIN(SLAIN) T 45.
801 AC=ASIN(SLAIN*SHEBO) T 52.
810 CONTINUE T 53.
C USE BOHEAD TO RESOLVE QUADRANT AMBIGUITY OF OMEGA AND ASNODE
1F(BOHEAD-90.0)8II,811,805 T 54.
805 IF(BOHEAD-180.0)806,806,807 T 55.
807 IF(BOHEAD-270.0)806,806,811 T 56.
811 OMEGA=-VEE(1)+AB T 57.
ASNOD=BOLONG+AC T 58.
GOTO804 T 59.
806 OMEGA=-VEE(1)-AB+PIE T 60.
ASNOD=BOLONG+AC+PIE T 61.
AR=PIF-AR T 61.
804 CONTINUE T 62.
COM=COS(OMEGA) T 63.
SOM=SIN(OMEGA) T 64.
C COMPUTE CONSTANT FACTORS IN ORBITAL POSITION EQUATIONS
C3=SOM*SINL T 65.
C4=COM*SINL T 66.
P=HBO*(1.+ECC*COS(VEE(1))) T 67.
VEE1=VEE(1)*CONVI T 68.
B=SORT(GMU)/P**1.5 T 69.
BETA=CONVI*OMEGA T 70.
SNODE=CONVI*ASNOD T 71.
PRINT 44,ECC,PLAIN,P,VEE1,BETA,SNODE T 71.
C COMPUTE TIME VARYING FACTORS IN ORBITAL POSITION EQUATIONS.
C LATITUDE, LONGITUDE, AND ALTITUDE OF SPACE VEHICLE RELATIVE TO
C MOVING EARTH
1=1 T 71.
1TIME=0 T 72.
700 CVEE=COS(VEE(1)) T 73.
SVEE=SIN(VEE(1)) T 74.
VEE1=VEE(1)*CONVI T 74.
TIME=1TIME T 75.
SLORE=SIN(BOLONG-WE*TIME) T 76.
CLORE=COS(BOLONG-WE*TIME) T 77.
X1=C1*CLORE-C2*SLORE T 78.
Y1=C2*CLORE+C1*SLORE T 79.
SLAR=C3*CVEE+C4*SVEE T 80.
CLAR=SORT(1.-SLAR**2) T 81.
C USE SHEBO TO RESOLVE PROGRADE AND RETROGRADE MOTIONS
11 IF(SLAR)766,777,777 T 82.
10 766 IF(SHEBO)56,55,55 T 83.
9   777 IF(SHEBO)55,56,56 T 84.
8   55 CONGR=ASNOD+ACOS((COM*CVEE-SOM*SVEE)/CLAR)-WE*TIME T 85.
     GOT057 T 86.
56 CONGR=ASNOD+ACOS((COM*CVEE-SOM*SVEE)/CLAR)-WE*TIME T 87.
57 CONTINUE T 88.
ALT=P/(1.+ECC*CVEE) T 89.
ALT1=ALT-RF T 90.
C REMOVE PERIODICITY EFFECTS FROM VEE
32 IF(VEE1-360.0)30,30,31 T 101.
31 VEE1=VEE1-360.0 T 102.
GO TO 32 T 103.
30 CONTINUE T 104.
CLOR=COS(CONGR) T 105.
SLOR=SIN(CONGR) T 106.
CNUMP=(1TIME-1TIME0) T 107.
RLAT=ASIN(SLAR)*CONVI T 108.
RLONG=CONGR*CONVI T 109.
JTIME=TIME/60. T 109.

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TIME=TIME/3600.0 T 110.
C   RESOLVE LONGITUDE INTO EAST AND WEST
36 IF(RLONG)33.34,35 T 111.
33 IF(RLONG+180.0)37.34,34 T 112.
37 RLONG=RLONG+360.0 T 113.
GO TO 36           T 114.
35 IF(RLONG-180.0)34.34,38 T 115.
38 RLONG=RLONG-360.0 T 116.
GO TO 36           T 117.
34 CONTINUE         T 118.
C   REMAINING STATEMENTS CONTROL RUNNING OF PROGRAM AND CALLING IN T 118.1
C   SUBROUTINES FOR INTEGRATION OF TRUE ANOMALY AND CALCULATION OF T 118.2
C   PARAMETERS BETWEEN STATION AND SATELLITE T 118.3
NIN2=NUM(2)          T 118.4
NIN4=NUM(4)          T 118.5
IF(NUM(7))701,701,720 T 119.
701 CALL CALC (NUM ,XI,YI,ZI ,RE,ITIME,CLAR,SLAR,CLOR,SLOR,RLAT, T 120.
2RLONG,ALTI,ALT,VEE1,BETA,SNODE) T 120.1
IF(NUM(7))721,721,720 T 120.2
720 IF(NPRINT+1)702,703,702
702 PRINT 61          T 120.4
NPRINT=-1            T 120.5
703 PRINT 65,ITIME,TIME,RLAT,RLONG,ALTI,VEE1 T 120.6
721 GO TO (722,714,715),1 T 121.
722 IF(NUM(6)) 724,724,723 T 122.
723 IF(ITIME-NUM(5))2,8,8 T 123.
724 IF(ITIME-NUM(3))2,3,3 T 124.
3 CHECK=ALT          T 125.
I=I+1                T 126.
GO TO 5               T 127.
714 IF(ALT-CHECK)6,6,5 T 128.
5 DO 4 J=1,NIN4       T 129.
CALL RUNG (VEE(1),B,ECC,NUM(8)) T 130.
4 CONTINUE             T 131.
ITIME=ITIME+NUM4      T 132.
GO TO 700              T 133.
6 I=I+1                T 134.
GO TO 2               T 135.
715 IF(ALT-RE)8,8,2    T 136.
2 DO 1 J=1,NIN2       T 137.
CALL RUNG (VEE(1),B,ECC,NUM(8)) T 138.
1 CONTINUE             T 139.
ITIME=ITIME+NUM2      T 140.
GO TO 700              T 141.
44 FORMAT(11X,29HORBITAL PARAMETERS AT BURNOUT/5X,12HECCENTRICITY,3X,T 141.1
211HINCLINATION,6X,12HSEM1 LAT REC,4X,12HTRUE ANOMALY,4X,5HOMEGA, T 141.2
36X,6HASNODE/24X,3HDEG,13X,4HN MI,13X,3HDEG,10X,3HDEG,9X,3HDEG/7X, T 141.3
4F7.6,7X,F7.2,9X,F8.2,9X,F7.2,6X,F7.2,5X,F8.3//) T 141.4
61 FORMAT(9X,4HTIME,5X,4HTIME,4X,69HVEHICLE LATITUDE VEHICLE LONGIT T 141.5
2UDE VEHICLE ALTITUDE TRUE ANOMALY/8X,14HMINUTES HOURS,10X,5HDET T 141.6
3G ,14X,3HDEG,16X,4HN MI,12X,3HDEG) T 141.7
65 FORMAT(8X,15,2X,F7.2,7X,F8.3,11X,F8.3,10X,F8.2,9X,F8.3) T 141.9
69 FORMAT(1HI,33X,50HSPACE VEHICLE TRACKING AND SENSOR COVERAGE PROGR T 141.90
2AM//) T 141.90
72 FORMAT(//11X,18HBURNOUT CONDITIONS/5X,8HATTITUDE,3X,8HVELOCITY, T 141.91
26X, 5HGAMMA,4X,7HHEADING,4X,8HLATITUDE,3X,9HLONGITUDE,4X,39H SPIN T 141.92
3AXIS ATTITUDE SPIN AXIS HEADING/7X,4HN MI,6X,6HFT/SEC,8X,3HDEG, T 141.93
47X,3HDEG,8X,3HDEG,9X,3HDEG,15X,3HDEG,18X,3HDEG/5X,F8.2,3X,F8.2, T 141.94
54X,F7.2,3X,F7.2,4X,F7.2,5X,F7.2,9X,F9.2,14X,F7.2//) T 141.95

```

B7 FORMAT(11X,25HPROGRAM CONTROL CONSTANTS/5X,118HNUM(1) NUM(2) NUM(3T 141.96  
2) NUM(4) NUM(5) NUM(6) NUM(7) NUM(8) NUM(9) NUM(10) NUM(11) NUM(12T 141.97  
3) NUM(13) NUM(14) NUM(15) NUM(16)/6X,13.4X,13.2X,17.2X,13.2X,17.3XT 141.98  
4,12.4X,13.3X,14.4X,13.4X,13.6(5X,13))  
T 141.99  
8 RETURN  
END  
T 142.  
T 143.



APPENDIX III  
FORTRAN IV LISTING  
SUBROUTINE CALC

LIST OF SYMBOLS USED IN SUBROUTINE CALC			
C	SYMBOL	UNITS	DEFINITION
C	ALAT	(DEG)	LATITUDE OF 1TH GROUND SENSOR
C	ALONG	(DEG)	LONGITUDE OF 1TH GROUND SENSOR
C	ALPHA	(DEG)	ANGLE BETWEEN LINE OF SIGHT AND 1TH FIXED GROUND SENSOR ANTENNA
C	ANAZ(1)	(DEG)	AZIMUTH OF 1TH GROUND SENSOR ANTENNA, IF FIXED
C	ANEL(1)	(DEG)	ELEVATION OF 1TH GROUND SENSOR ANTENNA, IF FIXED
C	ANGLE	(DEG)	ANGLE BETWEEN LINE OF SIGHT AND SPIN STABILIZED SPACE BORNE ANTENNA
C	AZD	(DEG)	AZIMUTH ANGLE OF LINE OF SIGHT BETWEEN 1TH GROUND SENSOR AND SPACE BORNE VEHICLE
C	BW1	(DEG)	BEAMWIDTH OF THE SPACEBORNE ANTENNA MEASURED BETWEEN HALF POWER POINTS
C	BW(1)	(DEG)	BEAMWIDTH OF 1TH GROUND SENSOR MEASURED BETWEEN HALF POWER POINTS
C	DBA(1)	(NONE)	DUMMY INPUT VARIABLE, SET EQUAL TO ON-AXIS GAIN OF THE 1TH GROUND STATION TRACKING ANTENNA
C	ELE	(DEG)	ELEVATION ANGLE OF LINE OF SIGHT BETWEEN 1TH GROUND SENSOR AND SPACEBORNE VEHICLE
C	ELEM(1)	(DEG)	MINIMUM ELEVATION ANGLE OF 1TH GROUND STATION TRACKING ANTENNA
C	FREQ	(MCS)	OPERATING FREQUENCY
C	FNGL	(DEG)	ANGLE BETWEEN LINE OF SIGHT AND EARTH POINTING SPACE BORNE ANTENNA
C	ITIME0	(SEC)	TIME AT PRECEEDING INTEGRATION INTERVAL
C	RHO	(N MI)	LINE OF SIGHT DISTANCE BETWEEN 1TH GROUND SENSOR
C	XAN(1)	(NONE)	DIRECTION COSINES OF 1TH GROUND SENSOR WITH FIXED ANTENNA AXIS
C	ZAN(1)		
C	XF	(NONE)	DIRECTIONAL COSINES FOR SPACE BORNE ANTENNA
C	YF		POINTING TOWARD EARTH CENTER
C	ZF		
C	THIS SUBROUTINE USES CALC TO MAKE IT COMPATIBLE WITH OTHER CALLING PROGRMS		
C	SUBROUTINE CALC (NUM,XI,YI,ZI ,RE,ITIME,CLAR,SLAR,CLOR,SLOR,RLAT,C 2RLONG,ALTI,ALT,VEE1,BETA,SNODE)		
C			1.1
C	DIMENSION ALAT(150),ALONG(150),CLA(150),SLA(150),CLO(150),SLO(150)C 2,ROH(150),EEL(150),ANGEL(150),ADZ(150),FNGL(150),ANEL(150),ANAZ(1C 350),XAN(150),YAN(150),ZAN(150),BW(150),NUM(16),DBA(150),ALPH(150)C 4,ALTS(150),ELEM(150),SNAME(2,150)		
C			1.2
C			1.3
C			1.4
C			1.401
C			1.41
C			1.42
C	CONV=0.01745329		
C			1.43
C	CONVI=57.29578		
C			1.44
C	CFEET=6076.115		
C			1.45
C	TIME=ITIME		
C			1.46
C	JTIME=TIME/60.0		
C			1.47
C	TIME=TIME/3600.0		
C			1.48
C	READ IN GROUND AND VEHICLE SENSOR DATA IF(MSTART+1)116,115,116		
C			1.49
116	MSTART=-1.		1.50
PRINT	73		1.51
READ	195,FREQ,BWI		1.52
195	FORMAT(2F10.2)		1.53

```

BWG=BW1 C 1.72
BW1=BW1*CONV
NUGS=NUM(1)
DO 101=1,NUGS
  READ 78, IDUM, (SNAME(J,I),J=1,2),ALAT(I),ALONG(I),ANEL(I),ANAZ/
    2ALTS(I),BW(I),ELEM(I)
 78 FORMAT (2X,13,3X,2A8 ,7F8.2) C 1.73
  PRINT75,I ,(SNAME(J,I),J=1,2),ALAT(I),ALONG(I),ANEL(I),ANAZ(I),C 1.8
    2ALTS(I),BW(I),ELEM(I) C 2.
 75 FORMAT (2X,13,3X,2A8 ,F7.2,2F9.2,IX,F9.2,3X,F9.2,1XC 3.0
    2,F9.2,9X,F7.2) C 3.1
  CLAS=CONV*ALAT(I) C 3.2
  CLOS=CONV*ALONG(I) C 3.25
  CLA(I)=COS(CLAS) C 3.3
  SLA(I)=SIN(CLAS) C 3.4
  CLO(I)=COS(CLOS) C 3.4
  SLO(I)=SIN(CLOS) C 3.4
  ANELR=CONV*ANEL(I) C 3.5
  ANAZR=CONV*ANAZ(I) C 3.6
  CANEL=COS(ANELR) C 3.7
  SANEL=SIN(ANELR) C 3.8
  CANAZ=COS(ANAZR) C 3.9
  SANAZ=SIN(ANAZR) C 3.9
  CA1=SANEL*CLA(I)-CANEL*CANAZ*SLA(I) C 4.0
  CA2=CANEL*SANAZ C 4.0
  XAN(I)=CA1*CLO(I)-CA2*SLO(I) C 4.1
  YAN(I)=CA2*CLO(I)+CA1*SLO(I) C 4.2
  ZAN(I)=SANEL*SLA(I)+CANEL*CANAZ*CLA(I) C 4.3
  10 CONTINUE C 4.4
  115 XF=CLAR*CLOR C 4.5
  YF=CLAR*SLOR C 4.6
  ZF=SLAR C 4.7
  M=0 C 4.8
  DO 62 L=1,NUGS C 4.9
    ALTSI=ALTS(L)/CFFFT C 5.0
    CPSI=SLA(L)*SLAR+CLA(L)*CLAR*(CLO(L)*CLOR+SLO(L)*SLOP) C 5.1
    RHO=SQRT(ALT**2+(RE+ALTSI)**2-ALT*(RE+ALTSI)*CPSI) C 5.2
    SELE=(ALT*CPSI-(RE+ALTSI))/RHO C 5.3
    CELE=SQRT(1.-SELE**2) C 5.4
    ELE=ASIN(SELE) C 5.5
  24 PHI=(SLAR-SLA(L)*CPSI)/(CLA(L)*SQRT(1.-CPSI**2)) C 5.6
    AZR=ACOS(PHI) C 5.7
    AZD=AZR*CONVI C 5.8
  C SWITCHING TO RESOLVE AMBIGUITY FOR EAST-WEST LAUNCHINGS IN NORTH C 5.9
  C AND SOUTH LATITUDE MODE C 6.0
    CLONGR=RLONG C 6.1
    CLONGA=ALONG(L) C 6.2
    IF(CLONGR-CLONGA)88.92,90 C 6.3
  12 88 IF(CLONGR-CLONGA+180.)192.92,93 C 6.4
  11 90 IF(CLONGR-CLONGA-180.)192.92,93 C 6.5
  10 93 AZD=360.0-AZD C 6.6
  * AZR=AZR*CONV C 6.7
  * 92 CONTINUE C 6.8
    CAZR=COS(AZR) C 6.9
    SAZR=SIN(AZR) C 6.10
    C5=SELE*CLA(L)-CELE*CAZR*SLA(L) C 6.11
    C6=CELE*SAZR C 6.12
    XRHO=C5*CLO(L)-C6*SLO(L) C 6.13
    YRHO=C6*CLO(L)+C5*SLO(L) C 6.14
    ZRHO=SELE*SLA(L)+CELE*CAZR*CLA(L) C 6.15
    ANGLE=ACOS(X1*XRH0+Y1*YRH0+Z1*ZRH0) C 6.16
    FNGLE=ACOS(XF*XRH0+YF*YRH0+ZF*ZRH0) C 6.17
    ALPHA=ACOS(XAN(L)*XRH0+YAN(L)*YRH0+ZAN(L)*ZRH0) C 6.18
    ELE=ELE*CONVI C 6.19

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ANGLE=ANGLE*CONVI C 35.
FNGLE=FNGLE*CONVI C 36.
ALPHA=ALPHA*CONVI C 37.
C CALCULATE AVERAGE RATE OF CHANGE OVER LAST PRINTOUT INTERVAL
CNUMB=ITIME-ITIMEO C 38.
IF(LSTART+1)151,150,151
150 DRHO=(RHO-ROH(L))/CNUMB*CFEET C 39.
DAZD=(AZD-ADZ(L))/CNUMB C 40.
DELE=(EEL-EEL(L))/CNUMB C 41.
DANGLE=(ANGLE-ANGEL(L))/CNUMB C 42.
DFNGLE=(FNGLE-FNGEL(L))/CNUMB C 43.
DALPHA=(ALPHA-ALPAH(L))/CNUMB C 44.
151 LSTART=-1 C 44.1
C SAVE VALUE OF VARIABLES AT PRECEEDING INTERVAL
ALPAH(L)=ALPHA C 45.
ROH(L)=RHO C 46.
EEL(L)=EEL C 47.
ANGEL(L)=ANGLE C 48.
FNGEL(L)=FNGLE C 49.
ADZ(L)=AZD C 50.
K=L C 51.
C ELIMINATE DATA WITH ELEVATIONS BELOW ACCEPTABLE MINIMUM
IF(ELE-ELEM(L))50,64,64 C 51.1
50 M=M+1 C 51.2
IF(NUCS M;51,51,62 C 51.3
51 ALPHA=0. C 51.31
ANGLE=0. C 51.32
FNGLE=0. C 51.33
IF(JSTART+1)64,30,64 C 52.
64 IF(NUM(9))111,111,112 C 52.
C CALL SUBROUTINE DEBE TO CALCULATE LOSS DATA IN DB
111 CALL DEBE(BW,ANGLE,ALPHA,FNGLE, BW1,RHO,K,DB8,SLOS, C 53.
2DB1,DB2,DB3,DB4,DB5,DB6,DB7,DB8,BWG,DBG,FREQ,GANGLE,GALPHA,GFNGLE) C 54.
112 CONTINUE C 54.1
IF(JSTART+1)141,140,141
141 JSTART=-1 C 54.4
PRINT 76,BWG,DBG,FREQ C 56.
PRINT 86 C 56.1
140 IF(KSTART+1)143,142,143 C 57.1
143 KSTART=-1 C 58.
30 PRINT 69,JTIME,TIME,RLAT,RLONG,ALTI,VEE1,BETA,SNODE C 58.
IF(NUGS-M)62,62,144 C 58.01
7
144 PRINT 85 C 58.1
PRINT 89 C 58.2
142 PRINT 60, (SNAME(J,L),J=1,2),ELE,AZD,RHO,ANGLE,FNGLE,ALPHA,GANGLE, 59.
2,GALPHA,DB1,DB3,DB5,DB7,DELE,DAZD,DRHO,DANGLE,DFNGLE,DALPHA,GFNGLFC 60.
3,SLOS,DB2,DB4,DB6,DB8 C 61.
62 CONTINUE C 62.
KSTART=+1 C 63.
60 FORMAT(2X,2A8,3X,F7.2,2X,F7.2,1X,F9.2,4X,F8.3,4X,F7.2,2X,F7.2,5X, C 64.
2F7.2,2X,F7.2,4(1X,F7.2,21X,F7.4,2X,F7.4,1X,E9.1,4X,F8.4,4X,F7.4,) C 65.
32X,F7.4,5X,F7.2,2X,F7.2,4(1X,F7.2,/) C 65.1
76 FORMAT(//10X,29HVEHICLE AND SENSOR PARAMETERS/5X,10HBEAM WIDTH,3X,C 66.
28HTRACKING,5X,9HOPERATING/BX, 3HDEG,9X,4HGAIN,7X,9HFREC 67.
3QUENCY /35X,2HMC /5X,F7.2,5X,F7.2,7X,F8.2) C 68.
73 FORMAT(11X,8HSTATIONS/1X,6HNUMBER,IX,4HNAME,15X,3HLAT,5X,4HLONG, C 70.
25X,4HELEV,5X,7HAZIMUTH,3X,8HALITUDE,3X,10HBEAM WIDTH,3X,14HMIN ELC 71.
3EV ANGLE/27X,3HDEG,6X,3HDEG,6X,3HDEG,7X,3HDEG,7X,4HFEET,BX, C 71.1
43HDEG,13X,3HDEG) C 71.2
86 FORMAT(//1X,62HTOTAL LOSSES FOR VARIOUS VEHICLE-GROUND ANTENNA CONC 72.
2FIGURATIONS C 73.

```

2	/5X,46HDB1=VEHICLE SPIN STABILIZED-GROUND FIXED	•	C 74.
2	/5X,46HDB2=VEHICLE SPIN STABILIZED-GROUND TRACKING	•	C 75.
2	/5X,46HDB3=VEHICLE EARTH CENTERED -GROUND FIXED	•	C 76.
2	/5X,46HDB4=VEHICLE EARTH CENTERED -GROUND TRACKING	•	C 77.
2	/5X,46HDB5=VEHICLE TRACKING-GROUND FIXED	•	
2	/5X,46HDB6=VEHICLE TRACKING-GROUND TRACKING	•	
2	/5X,46HDB7=VEHICLE SPIN STABILIZED-ISOTROPIC GROUND	•	C 80.
2	/5X,46HDB8=VEHICLE EARTH CENTERED-ISOTROPIC GROUND	•	C 81.
69	FORMAT( //1X,7HVEHICLE,4X,4HTIME,6X,4HTIME,4X,71HLATITUDE 2TUDE ALTITUDE TRUE ANOMALY OMEGA ASNODE/1X, 8HPOS1TC	LONGIC	82.
	31ON,2X,7HMINUTES,4X,5HHOURS,5X,3HDEG,10X,3HDEG,10X,2HNM,11X,3HDEG,C	83.	
	41OX,3HDEG,10X,3HDEG/12X,15,4X,F6,2,2X,F8,3,5X,F8,3,5X,F8,2,3(5X,F8C	84.	
	5,3))		C 85.1
85	FORMAT(/2X,8HSTAT1ONS,9X,9HELEVATION,2X,7HAZIMUTH,5X,5HRANGE,3X,9HC	86.	
	2ANT ANGLE,2X,9HANT ANGLE,2X,7HGRD ANT,1X,11HVEHICLE ANT,2X,7HGRD AC	87.	
	3NT,2X,29HVEHICLE-GROUND ANTENNA LOSSES/2X,2HIN,15X,9HELEV-RATE,2X,C	88.	
	47HAZ-RATE,1X,9HRANG RATE,3X,9HSPIR-STAB,2X,9HVERT-STAB,4X,5HANGLEC	88.1	
	5,4X,8HGA1NS DB,2X,7HGAIN DB/2X,7HCONTACT,16X,3HDEG,6X,3HDEG,3X,	C 88.2	
	67HNAUT M1,9X,3HDEG,8X,3HDEG,6X,3HDEG,3X,9HSPIR-STAB,4X,5HSPACE,	C 88.3	
	75X,3HDB1,5X,3HDB3,5X,3HDB5,5X,3HDB7 )	C 88.4	
89	FORMAT(21X,7HDEG/SEC,2X,7HDEG/SEC,2X,8HFEET/SEC,5X,7HDEG/SEC,4X, 27HDEG/SEC,2X,7HDEG/SEC, 3X,9HVERT-STAB,2X,7HLOSS DB,5X,3HDB2, 35X,3HDB4,5X,3HDB6,5X,3HDB8/)	C 88.5	
	1T1MEO=IT1ME	C 88.6	
	RETURN	C 88.7	
	END	C 94.	
		C 95.	
		C 96.	

APPENDIX IV  
FORTRAN IV LISTING  
SUBROUTINE RUNG

```

C THIS IS SUBROUTINE RUNG1
C THIS SUBROUTINE USES RUNG TO MAKE IT COMPATIBLE WITH OTHER
C CALLING PROGAMS
C
C      SUBROUTINE RUNG (VEE,B,ECC,NUMB)          R   1.
C      DIMENSION C(4),VEE(5),VEER(4)             R   2.
C
C      A MODIFIED FOURTH ORDER RUNGE KUTTA NUMERICAL INTEGRATION METHOD
C      IS USED IN THIS SUBROUTINE TO OBTAIN THE TRUE ANOMALY AS A
C      FUNCTION OF TIME IN SECONDS. NUMB IS INTEGRATION INTERVAL IN SECS.
C      IF(ISTART+I)104,103,104
C
104  C(1)=0.5
     C(2)=0.5
     C(3)=1.0
     C(4)=0.0
     TNUM=NUMB
     ISTART=-1
103  DO102K=1,4
     VEER(K)=B*(I.+ECC*COS(VEE(K)))**2*TNUM
102  VEE(K+1)=VEE(1)+VEER(K)*C(K)
101  VEE(1)=VEE(1)+(VEER(1)+(VEER(2)+VEER(3))*2.+VEER(4))/6.0
     RETURN
     END

```



APPENDIX V  
FORTRAN IV LISTING  
SUBROUTINE DEBE

---

T SUBTYPE FORTRAN LMAP

C LIST OF SYMBOLS USED IN SUBROUTINE DEBEI

C SYMBOL	C UNITS	C DEFINITION	D
G ANGLE	(DEG)	ANGLE BETWEEN SPIN-STABILIZED SPACEBORNE ANTENNA AND THE LINE OF SIGHT TO THE ITH GROUND STATION	D
C ALPHA	(DEG)	ANGLE BETWEEN THE AXIS OF THE FIXED ANTENNA OF THE ITH GROUND STATION AND THE VEHICLE LINE OF SIGHT	D
C BW	(DEG)	BEAMWIDTH OF THE ITH GROUND ANTENNA MEASURED BETWEEN THE HALF POWER POINTS	D
G BWG	(DEG)	DUMMY VARIABLE USED TO PRESERVE BW1	D
C BW1	(DEG)	BEAMWIDTH OF THE SPACEBORNE ANTENNA MEASURED BETWEEN HALF POWER POINTS	D
C DBA	(DB)	ON-AXIS GAIN OF THE ITH GROUND STATION TRACKING ANTENNA	D
C DBG	(DB)	ON-AXIS GAIN OF THE SPACE-BORNE TRACKING ANTENNA	D
C DB1	(DB)	TOTAL LOSS OF THE SPIN STABILIZED-GROUND FIXED ANTENNA CONFIGURATION	D
C DB2	(DB)	TOTAL LOSS OF THE SPIN STABILIZED-GROUND TRACKING ANTENNA CONFIGURATION	D
C DB3	(DB)	TOTAL LOSS OF THE EARTH CENTERED-GROUND FIXED ANTENNA CONFIGURATION	D
C DB4	(DB)	TOTAL LOSS OF THE EARTH CENTERED-GROUND TRACKING ANTENNA CONFIGURATION	D
C DB5	(DB)	TOTAL LOSS OF THE VEHICLE TRACKING-GROUND FIXED ANTENNA CONFIGURATION	D
C DB6	(DB)	TOTAL LOSS OF THE VEHICLE TRACKING-GROUND TRACKING ANTENNA CONFIGURATION	D
C DB7	(DB)	TOTAL LOSS OF THE SPIN STABILIZED-ISENTROPIC GROUND ANTENNA CONFIGURATION	D
C DB8	(NONE)	TOTAL LOSS OF THE EARTH CENTERED-ISENTROPIC GROUND ANTENNA CONFIGURATION	D
C FDSL	(DB)	FREQUENCY DEPENDENT PART OF SPACE LOSS	D
C FNGLLE	(DEG)	ANGLE BETWEEN EARTH CENTER POINTING SPACEBORNE ANTENNA AND LINE OF SIGHT OF ITH GROUND STATION	D
C FREQ	(MCS)	OPERATING FREQUENCY	D
C GALPHA	(DB)	GAIN OF THE FIXED ANTENNA OF ITH GROUND STATION	D
C GANGLE	(DB)	GAIN OF THE SPIN STABILIZED SPACE-BORNE ANTENNA	D
C GFNGLE	(DB)	GAIN OF THE EARTH CENTERED SPACE-BORNE ANTENNA	D
C L	(NONE)	STATION NUMBER OR INDEX	D
C RDSDL	(DB)	RANGE DEPENDENT PART OF SPACE LOSS	D
C RHO	(IN MI)	LINE OF SIGHT DISTANCE BETWEEN THE ITH GROUND STATION AND THE SPACE-BORNE VEHICLE	D
C SLOS	(DB)	TOTAL SPACE LOSS (FDSDL+RDSDL)	D
C THIS SUBROUTINE CALCULATES THE SPACE LOSSES AND THE GAINS OF			
C VARIOUS VEHICLE-GROUND STATION ANTENNA CONFIGURATIONS. ALL INPUTS			
C ARE PASSED THROUGH THE CALL STATEMENT IN SUBROUTINE CALC.			
C THIS SUBROUTINE USES DEBE TO MAKE IT COMPATIBLE WITH OTHER			
C CALLING PROGAMS			
C			
6	SUBROUTINE DEBE (BW,ANGLE,ALPHA,FNGLLE,	BW1,RHO,L,DBA,SLOS,	D 1.
7	2DB1,DB2,DB3,DB4,DB5,DB6,DB7,DB8,BWG(DBG,FREQ,GANGLE,GALPHA,GFNGLE)D	1.2	
8			

---

```

DIMENSION BW(150),DBA(150) D 2.
CONV = 0.01745329 D 3.
CON1=(3.54491/(BW(L)*CONV))**2*0.6 D 3.5
CON2=(3.54491/BW1)**2*0.8 D 3.6
FDSDL=20.*ALOG10((299.776*3.28083/6076.115)/FREQ) D 4.1
RDSDL=-20.*ALOG10(4.*3.1415927*RHO) D 4.2
DBG=10.*ALOG10(CON2) D 8.
DBA(L)=10.*ALOG10(CON1) D 11.
IF (ALPHA)125,124,125 D 13.
124 ARG =1.0 D 14.
GO TO 126 D 15.
125 AN1 =2.78332*ALPHA/BW(L) D 16.
ARG =(SIN(AN1)/AN1)**2 D 17.
126 GALPHA=10.*ALOG10(CON1*ARG) D 18.
IF (ANGLE)135,134,135 D 19.
134 ARG2 =1.0 D 20.
GO TO 136 D 21.
135 AN2 =2.78332*ANGLE/BWG D 22.
ARG2 =(SIN(AN2)/AN2)**2 D 23.
136 GANGLE=10.*ALOG10(CON2*ARG2) D 24.
IF (FNGLE)235,234,235 D 25.
234 ARG3 =1.0 D 26.
GO TO 236 D 27.
235 AN3 =2.78332*FNGLE/BWG D 28.
ARG3 =(SIN(AN3)/AN3)**2 D 29.
236 GFNGLE=10.*ALOG10(CON2*ARG3) D 30.
C ADD UP LOSSES IN DB FOR THE VARIOUS SYSTEM ANTENNA CONFIGURATIONS
SLOS =FDSDL+RDSDL D 31.
DB1 =SLOS+GANGLE+GALPHA D 32.
DB2 =SLOS+GANGLE+DBA(L) D 33.
DB3 =SLOS+GFNGLE+GALPHA D 34.
DB4 =SLOS+GFNGLE+DBA(L) D 35.
DB5 =SLOS+DBG+GALPHA D 36.
DB6 =SLOS+DBG+DBA(L) D 37.
DB7 =SLOS+GANGLE D 38.
DB8 =SLOS+GFNGLE D 38.1
RETURN D 39.
END D 40.

```

APPENDIX VI  
FORTRAN IV LISTING  
SUBROUTINES ASIN AND ACOS

T SUBTYPE.FORTRAN.LMAP

CASIN

---

```

FUNCTION ASIN (A)
DATA RAD (57.295779513)
IF (ABS(A)-1.) 1,2,2
2 IF (A) 20,21,21
20 IF (A+1.000001) 22,23,23
23 ASIN=-90./RAD
GO TO 10
22 IF (A+1.001) 24,25,25
25 ASIN=-90./RAD
PRINT 30
30 FORMAT(6H ARG OF ASIN SIGNIFICANTLY GT 1. ARG SET = 1 AND PROGR
2M CONTINUED)
GO TO 10
24 ASIN=-90./RAD
PRINT 31
31 FORMAT(6H ARG OF ASIN MUCH GREATER THAN ONE - PROBABLE PROGRAMMING
2ERROR)
GO TO 10
21 IF (A-1.000001) 43,43,42
43 ASIN=90./RAD
GO TO 10
42 IF (A-1.001) 45,45,44
45 ASIN=90./RAD
PRINT 30
GO TO 10
44 ASIN=90./RAD
PRINT 31
GO TO 10
1 ASIN=ATAN(A/SQRT(1.0-A**2))
10 RETURN
END

```

---

T SUBTYPE.FORTRAN.LMAP

ACOS

---

```

FUNCTION ACOS (A)
DATA RAD (57.295779513)
IF (ABS(A)-1.) 1,2,2
2 IF (A) 20,21,21
20 IF (A+1.000001) 22,23,23
23 ACOS=180./RAD
GO TO 10
22 IF (A+1.001) 24,25,25
25 ACOS=180./RAD
PRINT 30
30 FORMAT(6H ARG OF ACOS SIGNIFICANTLY GT 1. ARG SET = 1 AND PROGR
2M CONTINUED)
GO TO 10
24 ACOS=180./RAD
PRINT 31
31 FORMAT(6H ARG OF ACOS MUCH GREATER THAN ONE - PROBABLY PROGRAMMING

```

---

```
2 ERROR)
GO TO 10


---


21 IF (A-1.000001) 43,43,42
43 ACOS=0.0
GO TO 10
42 IF (A-1.001) 45,45,44
45 ACOS=0.0
PRINT 30
GO TO 10
44 ACOS=0.0
PRINT 31
GO TO 10
1 IF(A)6,5,7
6 ACOS=ATAN(SQRT(1.0-A**2)/A)+180.0/RAD
GO TO 10
7 ACOS=ATAN(SQRT(1.0-A**2)/A)
GO TO 10
5 ACOS=90.0/RAD
10 RETURN
END
```

---

## APPENDIX VII

### DERIVATION OF THE ANTENNA ASPECT ANGLES

To determine the space losses and antenna gains for different orientations of ground station and vehicle antenna axes, it is necessary to derive expressions for the angles between the antenna axes and selected space directions. In the DEBE subroutine, combinations of the following four distinct situations are considered.

- 1) Ground-station antenna fixed
- 2) Ground-station antenna tracking
- 3) Vehicle-antenna fixed
  - (a) Spin stabilized
  - (b) Stabilized on earth center
- 4) Vehicle-antenna tracking

The angle relations between selected space directions are simply obtained by resolving unit vectors along these directions into components along the X, Y, and Z axes of the reference earth-fixed coordinate system and by taking the dot product of the appropriate transformed unit vectors.

#### RESOLUTION OF FIXED GROUND-STATION ANTENNA

The direction of a fixed ground-station antenna is given by the azimuth (ANAZ) and elevation (ANEL) angle of the antenna axis relative to the local horizontal plane (see Fig. 15). Clockwise directions from north define positive azimuth angles and the up-direction define positive elevation angles. Two

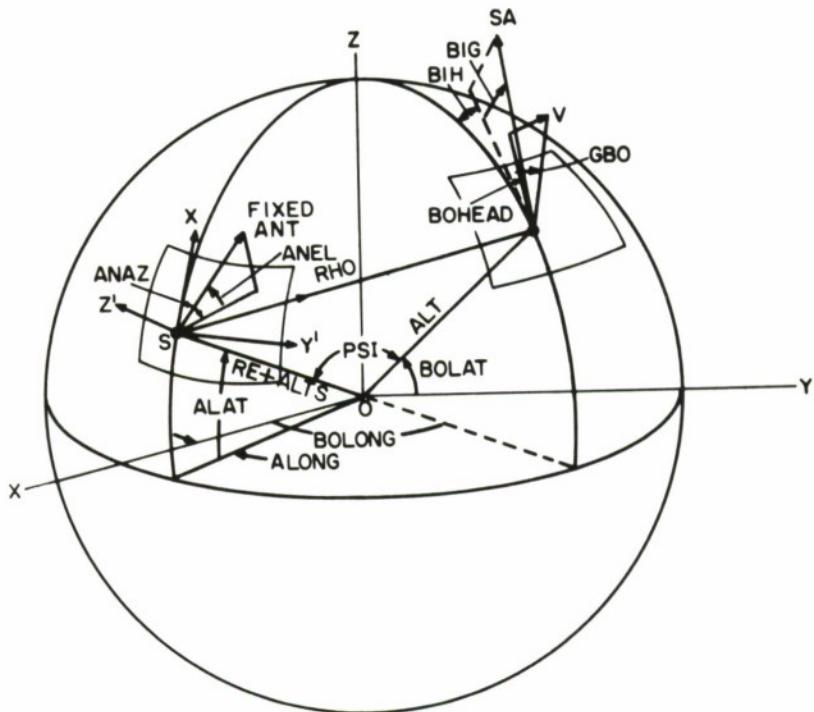


Fig. 15      Geometry for Unit-Vector Resolutions

resolutions transform a unit vector in this direction into components in earth-centered coordinates. The first resolves the vector into components along local tangent-plane axes:  $X'$  pointing north;  $Y'$  pointing west; and  $Z'$  pointing along the radius vector from the earth center. The second resolves these intermediate components into the components  $XAN$ ,  $YAN$ ,  $ZAN$  along the earth-centered axes. The orientation of the local tangent plane relative to the earth-centered axis system is defined by the geometric latitude ( $ALAT$ ) and longitude ( $ALONG$ ) of the station. The geometry between earth and antenna axis is shown in Fig. 15. The required resolutions are:

From the unit vector  $e_{AN}$  to components  $X'$ ,  $Y'$ ,  $Z'$  along local horizontal axes

$$\begin{vmatrix} X' \\ Y' \\ Z' \end{vmatrix} = \begin{vmatrix} \cos(ANEL) \cos(ANAZ) \\ \cos(ANEL) \sin(ANAZ) \\ \sin(ANEL) \end{vmatrix} \begin{vmatrix} e_{AN} \\ 0 \\ 0 \end{vmatrix} \quad (61)$$

From the local horizontal axes components  $X'$ ,  $Y'$ ,  $Z'$  to earth-centered components  $XAN$ ,  $YAN$ ,  $ZAN$ .

$$\begin{vmatrix} XAN \\ YAN \\ ZAN \end{vmatrix} = \begin{vmatrix} -\sin(ALAT) \cos(ALONG) & -\sin(ALONG) + \cos(ALAT) \cos(ALONG) \\ -\sin(ALAT) \sin(ALONG) & +\cos(ALONG) + \cos(ALAT) \sin(ALONG) \\ \cos(ALAT) & 0 & \sin(ALAT) \end{vmatrix} \begin{vmatrix} X' \\ Y' \\ Z' \end{vmatrix} \quad (62)$$

Combining these matrices gives the following components of the unit vector along the earth-centered axes.

$$\begin{aligned} XAN &= -\sin(ALAT) \cos(ALONG) \cos(ANEL) \cos(ANAZ) -\sin(ALONG) \\ &\quad \cos(ANEL) \sin(ANAZ) + \cos(ALAT) \cos(ALONG) \sin(ANEL) \\ YAN &= \sin(ALAT) \sin(ALONG) \cos(ANEL) \cos(ANAZ) + \cos(ALONG) \\ &\quad \cos(ANEL) \sin(ANAZ) + \cos(ALAT) \sin(ALONG) \sin(ANEL) \quad (63) \\ ZAN &= (\cos(ANEL) (\cos ANEZ) (\cos ALAT) + \sin(ANEL) \sin ALAT) \end{aligned}$$

## RESOLUTIONS FOR SPIN STABILIZED ANTENNA

The direction of the spin-stabilized antenna is given by the spin axis attitude at burnout (BIG) and the spin axis heading at burnout (BIH). Both of these angles are defined relative to the local horizontal plane with heading defined positive clockwise from north and attitude defined positive in the up direction. Two resolutions are required to transform the unit spin axis vector into components in earth-center coordinates. One resolves into the local tangent plane coordinates at the burnout condition: X' pointing north; Y' pointing east; and Z' pointing up. These intermediate components are then resolved into the earth-fixed axis components, XI, YI, ZI.

The axis system X', Y', Z' is fixed in inertia space and is defined by the burnout conditions, i.e., the latitude at burnout, BOLAT, and the longitude at burnout, BOLONG. Since the earth revolves, however, the longitude of an earth-fixed axis centered in the earth relative to axis fixed in inertia space will be (BOLONG - WE \* TIME) at any given time where (WE) (TIME), WET, is the longitude change of a point on the earth due to earth rotation. The required resolutions for the spin-stabilized unit vector are:

Resolution from unit vector  $e_I$  to local horizontal-plane (inertial) coordinates X', Y', Z'.

$$\begin{vmatrix} X' \\ Y' \\ Z' \end{vmatrix} = \begin{vmatrix} \cos BIG \cos BIH \\ \cos BIG \sin BIH \\ \sin BIG \end{vmatrix} \begin{vmatrix} e_I \\ 0 \\ 0 \end{vmatrix} \quad (64)$$

Resolution from local horizontal axes components  $X'$ ,  $Y'$ ,  $Z'$  to earth-centered axis components  $XI$ ,  $YI$ ,  $ZI$ .

$$\begin{pmatrix} XI \\ YI \\ ZI \end{pmatrix} = \begin{pmatrix} -\sin(BOLAT) \cos(BOLONG-WET) & -\sin(BOLONG-WET) \cos(BOLAT) \cos(BOLONG-WET) \\ -\sin(BOLAT) \sin(BOLONG-WET) & \cos(BOLONG-WET) \cos(BOLAT) \sin(BOLONG-WET) \\ \cos(BOLAT) & 0 & \sin(BOLAT) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (65)$$

$$\begin{aligned} XI &= -\sin(BOLAT) \cos(BOLONG-WET) \cos BIG \cos BIH \\ &\quad -\sin(BOLONG-WET) \cos BIG \sin BIH \\ &\quad +\cos(BOLAT) \cos(BOLONG-WET) \sin BIG \end{aligned}$$

$$\begin{aligned} YI &= -\sin(BOLAT) \sin(BOLONG-WET) \cos BIG \cos BIH \\ &\quad +\cos(BOLONG-WET) \cos BIG \sin BIH \\ &\quad +\cos(BOLAT) \sin(BOLONG-WET) \sin BIG \end{aligned} \quad (66)$$

$$ZI = \cos(BOLAT) \cos BIG \cos BIH + \sin(BOLAT) \sin BIG$$

If these components are arranged so that the time-dependent terms are collected, the expression becomes

$$\begin{aligned} XI &= \cos(BOLONG-WET) [\cos BOLAT \sin BIG \\ &\quad -\sin(BOLAT) \cos BIG \cos BIH] \\ &\quad -\sin(BOLONG-WET) [\cos BIG \sin BIH] \end{aligned}$$

$$\begin{aligned}
 YI = & \cos(\text{BOLONG-WET}) [\cos \text{BIG} \sin \text{BIH}] \\
 & + \sin(\text{BOLONG-WET}) [\cos \text{BOLAT} \sin \text{BIG} \\
 & - \sin \text{BOLAT} \cos \text{BIG} \cos \text{BIH}]
 \end{aligned} \tag{67}$$

$$ZI = \cos(\text{BOLAT}) \cos \text{BIG} \cos \text{BIH} + \sin(\text{BOLAT}) \sin \text{BIG}$$

The terms enclosed in the brackets are constant depending only on the burnout conditions BOLAT, BIG and BIH. They need be computed only once for any given set of initial conditions.

#### RESOLUTIONS FOR VEHICLE ANTENNA STABILIZED TO EARTH CENTER

In this configuration, the vehicle antenna is stabilized to point in the direction of the earth center during the orbit. The resolutions for this case are simply given in terms of the latitude and longitude at a particular time in the orbit.

One resolution transforms the unit vector  $e_F$  into components XF, YF, and ZF along the earth-centered axis.

$$\begin{vmatrix} X_F \\ Y_F \\ Z_F \end{vmatrix} = \begin{vmatrix} -\cos[\text{LATITUDE}(t_i)] \cos[\text{LONGITUDE}(t_i)] \\ -\cos[\text{LATITUDE}(t_i)] \sin[\text{LONGITUDE}(t_i)] \\ -\sin[\text{LATITUDE}(t_i)] \end{vmatrix} \begin{vmatrix} -e_F \\ 0 \\ 0 \end{vmatrix} \tag{68}$$

Where  $-e_1$  is a unit vector along a line through the earth center from the vehicle in orbit. Since the axis system in the vehicle is defined with the Z-axis pointing upward, the e-vector points in the negative direction.

$$\begin{aligned}
 X_F &= \cos(\text{RLAT}) \cos(\text{RLONG}) \\
 Y_F &= \cos(\text{RLAT}) \sin(\text{RLONG}) \quad (69) \\
 Z_F &= \sin(\text{RLAT})
 \end{aligned}$$

#### RESOLUTION OF LINE OF SIGHT FROM STATION TO VEHICLE (RHO)

The direction of line of sight to the vehicle is completely defined relative to the local tangent plane by the elevation angle (ELE) of the line of sight measured positive up from the horizontal plane and by the azimuth or heading angle (AZR) measured positive clockwise from north. As before, two resolutions are required to transform the unit vector along the line of sight into components along earth-fixed axes centered at the earth. The first resolution transforms the unit vector  $\bar{e}_{\text{RHO}}$  into components along the horizontal plane axes  $X'$ ,  $Y'$ , and  $Z'$ . The second resolution transforms this intermediate component into components  $X_{\text{RHO}}$ ,  $Y_{\text{RHO}}$ ,  $Z_{\text{RHO}}$  along the earth-fixed axes  $X$ ,  $Y$ ,  $Z$ .

The first resolution is from the unit vector  $e_{\text{RHO}}$  to components  $X$ ,  $Y$ ,  $Z$  along local horizontal-plane axes.

$$\begin{vmatrix} X' \\ Y' \\ Z' \end{vmatrix} = \begin{vmatrix} \cos(\text{ELE}) \cos \text{AZR} \\ \cos(\text{ELE}) \sin \text{AZR} \\ \sin(\text{ELE}) \end{vmatrix} \begin{vmatrix} e_{\text{RHO}} \\ 0 \\ 0 \end{vmatrix} \quad (70)$$

The second resolution is from local horizontal-axis components X', Y', and Z' to earth-centered components XRHO, YRHO, and ZRHO.

$$\begin{vmatrix} \text{XRHO} \\ \text{YRHO} \\ \text{ZRHO} \end{vmatrix} = \begin{vmatrix} -\sin(\text{ALAT}) \cos(\text{ALONG}) & -\sin(\text{ALONG}) + \cos(\text{ALAT}) \cos(\text{ALONG}) \\ -\sin(\text{ALAT}) \sin(\text{ALONG}) & \cos(\text{ALONG}) + \cos(\text{ALAT}) \sin(\text{ALONG}) \\ \cos(\text{ALAT}) & 0 & \sin(\text{ALAT}) \end{vmatrix} \begin{vmatrix} \text{X}' \\ \text{Y}' \\ \text{Z}' \end{vmatrix} \quad (71)$$

or combining these two matrices gives

$$\begin{aligned} \text{XRHO} = & -\sin(\text{ALAT}) \cos(\text{ALONG}) \cos(\text{ELE}) \cos(\text{AZR}) \\ & -\sin(\text{ALONG}) \cos(\text{ELE}) \sin(\text{AZR}) \\ & +\sin(\text{ELE}) \cos(\text{ALAT}) \cos(\text{ALONG}) \end{aligned}$$

$$\begin{aligned} \text{YRHO} = & -\sin(\text{ALAT}) \sin(\text{ALONG}) \cos(\text{ELE}) \cos(\text{AZR}) \\ & +\cos(\text{ALONG}) \cos(\text{ELE}) \sin(\text{AZR}) \\ & +\sin(\text{ELE}) \cos(\text{ALAT}) \sin(\text{ALONG}) \end{aligned} \quad (72)$$

$$\text{ZRHO} = \cos(\text{ALAT}) \cos(\text{ELE}) \cos(\text{AZR}) + \sin(\text{ELE}) \sin(\text{ALAT})$$

In these components the sines and cosines of ELE and AZR are the time-varying terms. ALAT and ALONG are fixed by the station location.

Finally, the cosine of the orientation angles between the station- and vehicle-antenna axes for the various configurations can be found by taking the appropriate dot products

$$\cos(\text{ALPHA}) = \bar{e}_{\text{RHO}} \cdot \bar{e}_{\text{AN}} = (\text{XRHO})(\text{XAN}) + (\text{YRHO}) \cdot (\text{YAN}) + (\text{ZRHO})(\text{ZAN}) \quad (73)$$

$$\cos(\text{ANGLE}) = \bar{e}_{\text{RHO}} \cdot \bar{e}_{\text{I}} = (\text{XRHO})(\text{XI}) + (\text{YRHO})(\text{YI}) + (\text{ZRHO})(\text{ZI}) \quad (74)$$

$$\cos(\text{FNGLE}) = \bar{e}_{\text{RHO}} \cdot \bar{e}_{\text{F}} = (\text{XRHO})(\text{XF}) + (\text{YRHO})(\text{YF}) + (\text{ZRHO})(\text{ZF}) \quad (75)$$

Where the components of the various unit vectors along the earth-fixed axis are given above in terms of vehicle and station position.



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13. ABSTRACT

This report is the second in a series describing the current effort towards establishing a workable engineering simulation of the space-ground environment. This report describes the simulation of ground station coverage of a vehicle from the end of powered flight to an arbitrary time in orbit. Signal strengths of several vehicle-ground station antenna combinations, as well as geometrical coverage, are simulated. A spherical earth model was used in the simulation, and the effects of atmospheric drag and orbital perturbations were neglected.

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